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# Mode-sum construction of the covariant graviton two-point function in the Poincaré patch of de Sitter space

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We construct the graviton two-point function for a two-parameter family of linear covariant gauges in  $n$ -dimensional de Sitter space. The construction is performed via the mode-sum method in the Bunch-Davies vacuum in the Poincaré patch, and a Fierz-Pauli mass term is introduced to regularize the infrared (IR) divergences. The resulting two-point function is de Sitter-invariant, and free of IR divergences in the massless limit (for a certain range of parameters) though analytic continuation with respect to the mass for the pure-gauge sector of the two-point function is necessary for this result. This general result agrees with the propagator obtained by analytic continuation from the sphere [Phys. Rev. D **34**, 3670 (1986); Class. Quant. Grav. **18**, 4317 (2001)]. However, if one starts with strictly zero mass theory, the IR divergences are absent only for a specific value of one of the two parameters, with the other parameter left generic. These findings agree with recent calculations in the Landau (exact) gauge [J. Math. Phys. **53**, 122502 (2012)], where IR divergences do appear in the spin-two (tensor) part of the two-point function. However, we find the strength (including the sign) of the IR divergence to be different from the one found in this reference.

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## I. INTRODUCTION

Quantum fields in de Sitter space have received increasing attention in recent years due to the accumulating experimental evidence for an inflationary epoch during the early moments of our Universe [1–5]. Moreover, the observations showing that the expansion of our Universe is accelerating suggest that it might attain a de Sitter stage in the future [6, 7]. On more theoretical grounds, the interest in de Sitter space stems from the fact that it is the maximally symmetric solution of Einstein equation with positive cosmological constant. Its isometries, very much like in the Minkowskian case, are expected to be reflected in the structure of the theory. This is indeed the case for massive scalar fields [8–11].

The analysis of gravitons propagating on the de Sitter geometry in this theoretical context is particularly relevant since their observable implications can serve as a probe to the inflationary era as well as a window to low-energy quantum gravity. The question of the existence of a state for free gravitons in de Sitter space that shares the background symmetries, however, is still a matter of contention in the literature. This thirty-year long controversy stems mainly from the fact that the natural graviton modes in the spatially flat coordinate patch of the de Sitter space (the Poincaré patch), which is the part relevant for inflationary cosmology, resemble those of a massless, minimally coupled (MMC) scalar field [12]. As is well-known, the natural vacuum state (the Bunch-Davies state) is IR-divergent for the MMC

scalar, and no de Sitter-invariant state exists [11, 12]. (Note, however, that there is a unitary representation of the de Sitter group corresponding to the free MMC scalar field [13–18] and that one can realize this representation by removing the mode responsible for the IR divergence [19].) The same is true for other cosmological (Friedman-Lemaître-Robertson-Walker, FLRW) spacetimes, of which the Poincaré patch of de Sitter space is a special case. The similarities between gravitons and scalar field modes led Ford and Parker to conclude that in all these spacetimes the graviton two-point function is IR-divergent, although the divergences they report do not appear in the physical quantities they studied [20].

A noteworthy distinction between the scalar field and linearized gravity is that the latter possesses gauge symmetries arising from the diffeomorphism invariance of general relativity. Therefore, it is important to settle whether these IR divergences are restricted to the gauge sector of linearized gravity, or they appear also in the physical sector, the exact definition of which is the source of disagreement in the literature. This question was addressed in Refs. [21, 22], where it was shown that in the traceless-transverse-synchronous gauge the IR-divergent part of the graviton two-point function can be expressed in a non-local pure-gauge form. The discussion was taken further by the authors of Ref. [23], who observed that a local gauge transformation on the graviton modes is sufficient to eliminate the IR divergences plaguing the graviton two-point function in that gauge. Moreover, examples of other gauges and coordinate systems in which the graviton two-point function is IR-finite have been worked out [24–30].

Another interesting test for the gauge nature of these IR divergences is provided by the linearized Weyl tensor, which is a local and gauge-invariant observable in

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the linearized theory. It was shown that the two-point function of the linearized Weyl tensor is IR-finite even if computed using a de Sitter-noninvariant graviton two-point function with an IR cutoff [31, 32], and that the result agrees with the one of Ref. [33], which was calculated from the covariant two-point function [25, 27, 28]. Moreover, it was shown that this two-point function is IR-finite also in slow-roll inflationary FLRW spacetimes, where the IR divergences of the graviton two-point functions are usually worse than in exact de Sitter space, as long as the slow-roll parameter is not too large [34]. Thus, the finiteness of the two-point function of the linearized Weyl tensor is not an accident due to the maximal symmetry of de Sitter space.

The covariant graviton two-point function was also shown to be physically equivalent to the transverse-traceless-synchronous one constructed on global de Sitter space [35] (in the sense that they produce the same two-point function of any local gauge-invariant tensor linear in the graviton field), and the latter is known to be IR-finite [29]. Therefore, it does not come as a surprise that within the gauge-invariant formulation of linearized gravity of Ref. [36], the graviton and the Weyl-tensor two-point functions were shown to contain the same gauge-invariant information in the Poincaré patch [37]. Furthermore, the existence of a de Sitter-invariant Hadamard state for the graviton was recently verified in Ref. [38], although the definition of such a state in this paper is technically different from the one originally proposed in Ref. [36].

Nevertheless, over the years many authors have employed IR-regulated graviton propagators which explicitly break de Sitter invariance in computing loop corrections. From these studies it has been claimed, e.g., that the cosmological constant decreases in time [39–41] (see Refs. [42, 43] for a criticism of these results and the rebuttal), that geometrical fluctuations can grow in the IR in inflationary spacetimes [44–46], and that some coupling constants change in time [47, 48]. Overall, these results suggest that the IR sector of the interacting theory could harbor important physical effects which break de Sitter invariance. They are rather intriguing, nonetheless, given the pure-gauge form of the de Sitter-breaking terms in the free propagator, on which those effects rely. Thus, loop calculations starting from de Sitter-invariant graviton propagators would be important to decide whether or not those effects are gauge artifacts.

However, as mentioned above, there are still some objections in the literature concerning the derivation of de Sitter-invariant graviton propagators [49]. Indeed, a graviton two-point function that breaks de Sitter invariance was obtained recently in the so-called exact de Donder gauge [50–52] by means of a formalism of covariant projection operators acting on scalar quantities. Then, in a follow up paper, Morrison took advantage of some freedom in the solutions of the differential equations satisfied by those scalar quantities in that formalism and constructed a de Sitter-invariant graviton two-point func-

tion, which is equivalent to the de Sitter-noninvariant one when smeared with transverse-traceless tensors of compact support [38], in order to extract the gauge-invariant content [36]. This work of Morrison’s was criticized in Ref. [53], with the main criticism being that the freedom argued to occur in Ref. [38] is not actually present if one were to derive the two-point function from a mode sum.

In this paper we revisit the question of the existence of a de Sitter-invariant state for gravitons, performing canonical quantization of the graviton field in the Poincaré patch of the de Sitter space and then constructing the corresponding two-point function via mode sums. We consider a two-parameter family of covariant linear gauges with a Fierz-Pauli mass term [54] which serves as an IR regulator. We then study the IR behavior of the two-point function thus obtained and analyze the convergence of the mode sums when we take the mass to zero after performing the momentum integrals [and after analytic continuation with respect to the mass in the pure-gauge vector (spin-1) sector]. Note that while in flat space the van Dam-Veltman-Zakharov discontinuity prevents this limit from being smooth [55, 56], there is no such problem in de Sitter space [21, 57], at least in the linear regime. We also consider the result obtained by taking the zero mass limit before the momentum integration. We recover the de Sitter-invariant and IR-finite results of Ref. [38] if we take the massless limit after the momentum integration. Interestingly, if we use the procedure of taking the massless limit before performing the momentum integrals, the result differs from both the IR-divergent and IR-finite results in the literature. We do agree with the IR-divergent results [50–52] in the sense that for their choice of gauge parameters IR divergences appear in the tensor (spin-2) sector. However we disagree even on the sign of the IR divergence. Our complete results reveal that there exists a de Sitter-invariant, IR-finite graviton two-point function in the massless limit for a generic choice of gauge parameters if this limit is taken after the momentum integrals (and after a certain analytic continuation in the pure-gauge sector) and for a particular value of one of the gauge parameters if it is taken before the momentum integrals.

The rest of the paper is organized as follows: we start in Sec. II A with a brief discussion on the canonical quantization of a general free field theory via the symplectic product method. In Sec. II B we present the Lagrangian density for the graviton, with the mass and gauge-fixing terms in addition to the gauge-invariant part, and decompose the field in its scalar, vector, and transverse-tensor sectors. We canonically quantize each sector with the aid of symplectic product method in Secs. III A, III B, and III C and calculate the corresponding two-point functions through their mode-sum definitions. The massless limit of the two-point function thus obtained is then studied in Sec. III D, and its behavior for large separations is obtained in Sec. III E. The convergence of the momentum integrals in the mode sums in the IR (with the massless limit taken before or after the momentum integration)

is investigated in detail in Sec. IV. We summarize and discuss our results and make some remarks on them in Sec. V. Throughout this paper we use units such that  $\hbar = c = 1$ , and use the mostly plus convention for the metric.

## II. CANONICAL QUANTIZATION

### A. Overview of the symplectic product method

Let us consider a free field theory described by a Lagrangian density  $\mathcal{L}$  which is defined on a spacetime with background metric  $g_{ab}$  and is a function of the symmetric tensor field  $h_{ab}$  and its covariant derivative  $\nabla_c h_{ab}$ . The canonical conjugate momentum current  $p^{abc}$  is defined from  $\mathcal{L}$  by

$$p^{abc} \equiv \frac{1}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial (\nabla_a h_{bc})}, \quad (1)$$

where  $g$  is the determinant of the background metric. The Euler-Lagrange equation for  $h_{ab}$  can then be written as

$$\nabla_a p^{abc} - \frac{1}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial h_{bc}} = 0. \quad (2)$$

For any two solutions  $h_{ab}^{(1)}$  and  $h_{ab}^{(2)}$  of Eq. (2), it follows that the current

$$J^{(1,2)a} \equiv -i \left( h_{bc}^{(1)*} p^{(2)abc} - h_{bc}^{(2)} p^{(1)abc*} \right) \quad (3)$$

(with a star denoting complex conjugation) is conserved, i.e.,  $\nabla_a J^{(1,2)a} = 0$ . Hence, the symplectic product

$$\langle h_{ab}^{(1)}, h_{cd}^{(2)} \rangle \equiv \int_{\Sigma} J^{(1,2)a} n_a d\Sigma \quad (4)$$

with  $\Sigma$  a Cauchy surface,  $n_a$  the future-directed unit normal to it, and  $d\Sigma$  the normalized surface element, is time-independent and thus independent of the choice of  $\Sigma$ .

If the symplectic product (4) is non-degenerate, i.e., if there is no solution with zero symplectic product with all solutions, we quantize the classical field  $h_{ab}$  by imposing the usual canonical commutation relations on the field operator  $\hat{h}_{ab}$  and its canonical conjugate momentum  $n_a \hat{p}^{abc}$ :

$$\left[ \hat{h}_{ab}(\mathbf{x}), n_c \hat{p}^{ecd}(\mathbf{x}') \right] \Big|_{\Sigma} = -i \delta_a^c \delta_b^d \delta(\mathbf{x}, \mathbf{x}') \quad (5)$$

with all other equal-time commutators vanishing. The  $\delta$  distribution appearing here is the one associated with the spatial sections, defined by

$$\int \delta(\mathbf{x}, \mathbf{x}') f(\mathbf{x}') N(\mathbf{x}') d\Sigma' = f(\mathbf{x}), \quad (6)$$

where  $N = n_a (\partial/\partial t)^a$  is the lapse function, with  $t$  parametrizing the Cauchy surfaces and increasing towards the future. In order to construct a representation

for the field operator one then chooses a complete set of modes  $\{h_{ab}^{(k)}\}_{k \in \mathcal{I}}$  and  $\{h_{ab}^{(k)*}\}_{k \in \mathcal{I}}$  of the Euler-Lagrange equation (2), with  $\mathcal{I}$  an appropriate set of quantum numbers, such that  $h_{ab}^{(k)}$  and  $h_{ab}^{(k')*}$  have vanishing symplectic product. Then one expands the quantum field  $\hat{h}_{ab}$  in terms of this complete set of modes as

$$\hat{h}_{ab} = \sum_{k \in \mathcal{I}} \left( \hat{A}_k h_{ab}^{(k)} + \hat{A}_k^\dagger h_{ab}^{(k)*} \right). \quad (7)$$

From the symplectic product (4) and the commutation relations (5), it is straightforward to show that

$$\left[ \langle h_{ab}^{(k)}, \hat{h}_{cd} \rangle, \langle \hat{h}_{ab}, h_{cd}^{(l)} \rangle \right] = \langle h_{ab}^{(k)}, h_{cd}^{(l)} \rangle \equiv M_{kl}, \quad (8)$$

where the (Hermitian) matrix  $M_{kl}$  is invertible, thanks to the completeness of our set of modes. On the other hand, by employing the expansion (7) one obtains

$$\left[ \langle h_{ab}^{(k)}, \hat{h}_{cd} \rangle, \langle \hat{h}_{ab}, h_{cd}^{(l)} \rangle \right] = \sum_{k', l' \in \mathcal{I}} M_{kk'} \left[ \hat{A}_{k'}, \hat{A}_{l'}^\dagger \right] M_{l'l}. \quad (9)$$

Therefore, one concludes that the operators  $\hat{A}_k$  and  $\hat{A}_k^\dagger$  must satisfy the following commutation relations:

$$[\hat{A}_k, \hat{A}_l^\dagger] = (M^{-1})_{kl}. \quad (10)$$

By a similar calculation with  $h_{cd}^{(l)}$  replaced by  $h_{cd}^{(l)*}$  we find

$$[\hat{A}_k, \hat{A}_l] = 0. \quad (11)$$

Thus, in the state  $|0\rangle$  annihilated by all the  $\hat{A}_k$ , the Wightman two-point function,

$$\Delta_{abc'd'}(x, x') \equiv \langle 0 | \hat{h}_{ab}(x) \hat{h}_{c'd'}(x') | 0 \rangle, \quad (12)$$

is given by the mode sum

$$\Delta_{abc'd'}(x, x') = \sum_{k, l \in \mathcal{I}} (M^{-1})_{kl} h_{ab}^{(k)}(x) h_{c'd'}^{(l)}(x'), \quad (13)$$

where the primed indices refer to quantities defined on the point  $x'$  here and below. For a more detailed exposition of this method, which is completely equivalent to the usual canonical quantization scheme but is technically easier to use if the field equations for  $\hat{h}_{ab}$  are complicated, see, e.g., Ref. [57].

### B. The Lagrangian

We start from the Einstein-Hilbert Lagrangian density for gravity plus a positive cosmological constant  $\Lambda$  in an  $n$ -dimensional spacetime,

$$\mathcal{L}_{\text{grav}} \equiv \frac{1}{\kappa^2} (\bar{R} - 2\Lambda) \sqrt{-\bar{g}}, \quad (14)$$

where  $\kappa^2 \equiv 16\pi G_N$  with Newton's constant  $G_N$ ,  $\bar{g}_{ab}$  denotes the full metric (background plus perturbations), and  $\bar{R}$  and  $\bar{g}$  are the corresponding scalar curvature and metric determinant, respectively. We take the background metric  $g_{ab}$  to be the de Sitter metric, which in the Poincaré patch reads

$$g_{ab} = a(\eta)^2 \eta_{ab}, \quad (15)$$

with  $\eta_{ab}$  the flat (Minkowski) metric,  $a(\eta) \equiv (-H\eta)^{-1}$  the conformal factor,  $\eta \in (-\infty, 0)$  the conformal time, and  $H$  the Hubble constant.

We write the full metric as  $\bar{g}_{ab} = g_{ab} + \kappa h_{ab}$  and expand the Lagrangian (14) up to second order in the perturbation  $h_{ab}$ . Choosing the cosmological constant to be  $2\Lambda = (n-1)(n-2)H^2$ , one finds that the part of the Lagrangian density linear in the metric perturbations vanishes up to a boundary term. The second-order part can then be cast, up to a boundary term, into the form

$$\begin{aligned} \mathcal{L}_{\text{inv}} \equiv & -\frac{1}{4} \left[ \nabla_c h_{ab} \nabla^c h^{ab} - \nabla_c h \nabla^c h \right. \\ & + 2\nabla_a h^{ab} \nabla_b h - 2\nabla^a h_{ac} \nabla_b h^{bc} \\ & \left. + 2H^2 \left( h_{ab} h^{ab} + \frac{n-3}{2} h^2 \right) \right] \sqrt{-g}, \end{aligned} \quad (16)$$

where we have used that the Riemann tensor of de Sitter space is  $R_{abcd} = H^2(g_{ac}g_{bd} - g_{ad}g_{bc})$ . As usual,  $h \equiv h^a_a$  denotes the trace of  $h_{ab}$ , and indices are raised and lowered with the background metric  $g_{ab}$ . It can readily be verified that  $\mathcal{L}_{\text{inv}}$  is invariant (up to a boundary term) under the gauge transformation

$$h_{ab} \rightarrow h_{ab} - \nabla_a \xi_b - \nabla_b \xi_a \quad (17)$$

for any vector field  $\xi^a$ .

As is well known, because of this gauge symmetry a direct quantization of  $\mathcal{L}_{\text{inv}}$  is not possible. The reason is that the symplectic product (4) between a pure gauge mode and any other solution of the linearized Einstein equation is identically zero [58], which implies that  $M$  is a degenerate matrix and that the inverse  $M^{-1}$  in Eq. (10) does not exist. This difficulty can be circumvented by adding a gauge-fixing term in the Lagrangian density in such a way that  $M$  becomes an invertible matrix. In our case, we add the following most general linear covariant

gauge-fixing term to the Lagrangian density:

$$\mathcal{L}_{\text{gf}} \equiv -\frac{1}{2\alpha} G_b G^b \sqrt{-g}, \quad (18)$$

with

$$G_b \equiv \nabla^a h_{ab} - \frac{1+\beta}{\beta} \nabla_b h, \quad (19)$$

where  $\alpha$  and  $\beta$  are real parameters. The analogue of the Feynman gauge ( $\xi = 1$ ) in electromagnetism, for which the gauge-fixing term is  $-1/(2\xi)(\nabla_a A^a)^2$ , is achieved for  $\alpha = 1$  and  $\beta = -2$ , while a one-parameter family analogous to the Landau gauge can be obtained in the limit  $\alpha \rightarrow 0$ . (These gauges are called the exact gauge, and, for the special case  $\beta = -2$ , the exact de Donder gauge in Refs. [50, 51].) However, these gauges may not always be the most useful ones. An example of a less well-known but useful gauge from electromagnetism is the Fried-Yennie gauge, a covariant gauge with gauge parameter  $\xi = (n-1)/(n-3)$  [59]. This gauge makes individual Feynman diagrams IR-finite (as opposed to only their sum), and is extremely useful in bound-state calculations [60]. Note that our  $\beta$  is the same as the one used in Refs. [27, 28], while it is related to the parameter  $b$  used in Ref. [52] (for which we have to take in addition  $\alpha = 0$ ) by

$$\beta = \frac{2}{b-2}. \quad (20)$$

We also introduce in the Lagrangian density the Fierz-Pauli mass term [54],

$$\mathcal{L}_{\text{mass}} \equiv -\frac{m^2}{4} (h_{ab} h^{ab} - h^2) \sqrt{-g}, \quad (21)$$

which will serve to regulate the IR behavior of the theory. We will take the  $m \rightarrow 0$  limit in the end.

The total Lagrangian density

$$\mathcal{L} \equiv \mathcal{L}_{\text{inv}} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{mass}} \quad (22)$$

leads to the field equation

$$L_{ab}^{(\text{inv})cd} h_{cd} + L_{ab}^{(\text{gf})cd} h_{cd} - m^2 (h_{ab} - g_{ab} h) = 0, \quad (23)$$

where we have defined the differential operators  $L_{ab}^{(\text{inv})cd}$  and  $L_{ab}^{(\text{gf})cd}$  by

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$$L_{ab}^{(\text{inv})cd} h_{cd} \equiv 2 \int \frac{\delta \mathcal{L}_{\text{inv}}}{\delta h^{ab}} d^n x = \nabla^2 h_{ab} + g_{ab} (\nabla_c \nabla_d h^{cd} - \nabla^2 h) + \nabla_a \nabla_b h - 2\nabla_{(a} \nabla^c h_{b)c} - H^2 [2h_{ab} + (n-3)g_{ab}h], \quad (24)$$

with the abbreviation  $\nabla^2 = \nabla_a \nabla^a$ , and

$$L_{ab}^{(\text{gf})cd} h_{cd} \equiv 2 \int \frac{\delta \mathcal{L}_{\text{gf}}}{\delta h^{ab}} d^n x = \frac{2}{\alpha} \left( \nabla_{(a} G_{b)} - \frac{1+\beta}{\beta} g_{ab} \nabla_c G^c \right), \quad (25)$$

with  $G_b$  defined by Eq. (19). For the total Lagrangian density, Eq. (22), the canonical conjugate momentum current defined by Eq. (1) can be written in the form

$$p^{abc} = p_{\text{inv}}^{abc} + p_{\text{gf}}^{abc}, \quad (26)$$

with

$$p_{\text{inv}}^{abc} \equiv -\frac{1}{2} \left[ \nabla^a h^{bc} + g^{a(b} (\nabla^{c)} h - 2 \nabla_d h^{c)d} \right. \\ \left. + g^{bc} (\nabla_d h^{ad} - \nabla^a h) \right] \quad (27)$$

coming from the gauge-invariant Lagrangian density  $\mathcal{L}_{\text{inv}}$ , Eq. (16), and with

$$p_{\text{gf}}^{abc} \equiv -\frac{1}{\alpha} \left( g^{a(b} G^{c)} - \frac{1+\beta}{\beta} g^{bc} G^a \right) \quad (28)$$

following from the gauge-fixing term  $\mathcal{L}_{\text{gf}}$ , Eq. (18).

In order to obtain the two-point function, we have to find a complete set of modes  $h_{ab}^{(k)}$  that satisfy Eq. (23). This task is facilitated by decomposing the metric perturbation into (covariant) scalar, vector, and tensor sectors as

$$h_{ab} = h_{ab}^{(S)} + h_{ab}^{(V)} + h_{ab}^{(T)}. \quad (29)$$

The scalar sector is defined in terms of two scalars,  $B$  and  $\Psi$ , as

$$h_{ab}^{(S)} \equiv \nabla_a \nabla_b B + g_{ab} \Psi, \quad (30)$$

the vector sector is given by

$$h_{ab}^{(V)} \equiv \nabla_a v_b + \nabla_b v_a, \quad (31)$$

with the vector field  $v_a$  satisfying the divergence-free condition,  $\nabla_a v^a = 0$ , and the tensor sector is transverse and traceless, i.e.,  $h^{(T)} = \nabla^a h_{ab}^{(T)} = 0$ . It is here that the symplectic product method shows its advantages, as it is not necessary to determine the canonical conjugate momentum for each of these sectors taking into account these constraints, which would be a tedious task. Instead, one can simply restrict the canonical conjugate momentum current (26) to each of the sectors. Since the scalar, vector, and tensor sectors are orthogonal to each other with respect to the symplectic product (4), the matrix  $M$  defined by Eq. (8) becomes block-diagonal and we can study each sector separately. [This is of course equivalent to the fact that the three sectors decouple in the Lagrangian (22)].

### III. THE TWO-POINT FUNCTION

#### A. Scalar sector

We begin the construction of the field modes in the scalar sector. For that sector, the field equation (23) can be cast in the form

$$\nabla_a \nabla_b \Phi_1 + g_{ab} \Phi_2 = 0, \quad (32)$$

with  $\Phi_1$  and  $\Phi_2$  defined by

$$\Phi_1 \equiv -\frac{2}{\alpha\beta} \left[ \nabla^2 - (n-1)\beta H^2 + \frac{\alpha\beta}{2} m^2 \right] B \\ + \frac{(n-2)\alpha\beta - 2n - 2(n-1)\beta}{\alpha\beta} \Psi \quad (33)$$

and

$$\Phi_2 \equiv -(n-2)\nabla^2 \Psi + \frac{2(1+\beta)[n + (n-1)\beta]}{\alpha\beta^2} \nabla^2 \Psi \\ + \frac{2(1+\beta)}{\alpha\beta^2} \left[ \nabla^2 - (n-1)\beta H^2 + \frac{\alpha\beta}{2} m^2 \right] \nabla^2 B \\ + (n-1)[m^2 - (n-2)H^2] \Psi - \frac{1}{\beta} m^2 B. \quad (34)$$

Obviously, Eq. (32) is satisfied if  $\Phi_1 = \Phi_2 = 0$ , which leads to the equations

$$\left[ \nabla^2 - (n-1)\beta H^2 + \frac{\alpha\beta}{2} m^2 \right] B = -\left( n + \frac{\lambda\beta}{2} \right) \Psi \quad (35)$$

and

$$\left[ \nabla^2 - (n-1)\beta H^2 + \frac{\alpha\beta}{2} m^2 \right] \Psi \\ = -\frac{\beta m^2}{n-2} \left[ \frac{\alpha m^2}{2} - (n-1)H^2 \right] B - \frac{n+\lambda\beta}{n-2} m^2 \Psi, \quad (36)$$

where we have defined  $\lambda \equiv 2(n-1) - (n-2)\alpha$ . Due to the presence of the mass, Eqs. (35) and (36) are coupled and cannot be solved in a straightforward manner. Nevertheless, since we are interested in the massless limit and since the scalar sector is well-behaved in the IR if  $\beta > 0$  (as noted before, see Refs. [27, 28, 52]), we make this assumption for  $\beta$  and let  $m = 0$  straight away. This leads to the equations

$$[\nabla^2 - (n-1)\beta H^2] B = -\left( n + \frac{\lambda\beta}{2} \right) \Psi \quad (37)$$

and

$$[\nabla^2 - (n-1)\beta H^2] \Psi = 0, \quad (38)$$

which are coupled Klein-Gordon equations with squared mass  $M_\beta^2 \equiv (n-1)\beta H^2$ . One can prove that, if  $h_{ab}$  satisfies the field equation (23), then the trace  $h$  and the scalar  $\nabla^a \nabla^b h_{ab}$  can be reproduced by  $h_{ab}^{(S)}$  of the form (30) with  $B$  and  $\Psi$  satisfying Eqs. (37) and (38). This implies that one can write  $h_{ab} = h_{ab}^{(V+T)} + h_{ab}^{(S)}$ , where  $h_{ab}^{(V+T)}$  is traceless and satisfies  $\nabla^a \nabla^b h_{ab}^{(V+T)} = 0$ .

The scalar sector of our two-point function will be expressed in terms of that of the Klein-Gordon scalar field of squared mass  $M_\beta^2$ . It is known that the scalar-field two-point function is well-behaved in the IR if and only

if the mass is strictly positive in the Poincaré patch. For this reason we require  $\beta > 0$  as we stated before (and exclude, e.g., the analogue of the Feynman gauge).

In the Poincaré patch, the Klein-Gordon equation for a scalar field  $\phi$  with mass  $M$  reads

$$\left(\partial_\eta^2 - \frac{n-2}{\eta}\partial_\eta - \Delta + \frac{M^2}{H^2\eta^2}\right)\phi = 0, \quad (39)$$

where  $\Delta \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$  is the flat-space Laplace operator, and admits mode solutions of the form

$$\phi_{\mathbf{p}}^{M^2}(\eta, \mathbf{x}) \equiv f_{M^2}(\eta, \mathbf{p})e^{i\mathbf{p}\mathbf{x}}. \quad (40)$$

Here,  $f_{M^2}(\eta, \mathbf{p})$  is a function that is used extensively in this work and takes the following form:

$$f_{M^2}(\eta, \mathbf{p}) \equiv \sqrt{\frac{\pi}{4H}}(-H\eta)^{\frac{n-1}{2}}e^{i\mu\frac{\pi}{2}}H_\mu^{(1)}(-|\mathbf{p}|\eta) \quad (41)$$

with  $H_\mu^{(1)}$  the Hankel function of the first kind and the parameter

$$\mu \equiv \sqrt{\frac{(n-1)^2}{4} - \frac{M^2}{H^2}}. \quad (42)$$

Two linearly independent sets of solutions to Eqs. (37) and (38), which together with their complex conjugates form a complete set, can then be constructed from the mode solutions Eq. (40). In terms of these modes, these two linearly independent sets of solutions are

$$B_{\mathbf{p}}^{(1)}(\eta, \mathbf{x}) = \phi_{\mathbf{p}}^{M_\beta^2}(\eta, \mathbf{x}), \quad (43a)$$

$$\Psi_{\mathbf{p}}^{(1)}(\eta, \mathbf{x}) = 0, \quad (43b)$$

and

$$B_{\mathbf{p}}^{(2)}(\eta, \mathbf{x}) = -\left(n + \frac{\lambda\beta}{2}\right)\frac{\partial}{\partial M^2}\phi_{\mathbf{p}}^{M^2}(\eta, \mathbf{x})\Big|_{M^2=M_\beta^2}, \quad (44a)$$

$$\Psi_{\mathbf{p}}^{(2)}(\eta, \mathbf{x}) = \phi_{\mathbf{p}}^{M_\beta^2}(\eta, \mathbf{x}). \quad (44b)$$

Recalling the definition of  $h_{ab}^{(S)}$  in terms of  $B$  and  $\Psi$ , Eq. (30), we thus obtain the modes of the scalar sector:

$$h_{\mathbf{p}ab}^{(S,1)}(\eta, \mathbf{x}) \equiv \nabla_a\nabla_b\phi_{\mathbf{p}}^{M_\beta^2}(\eta, \mathbf{x}) \quad (45)$$

and

$$h_{\mathbf{p}ab}^{(S,2)}(\eta, \mathbf{x}) \equiv g_{ab}\phi_{\mathbf{p}}^{M_\beta^2}(\eta, \mathbf{x}) - \left(n + \frac{\lambda\beta}{2}\right)\nabla_a\nabla_b\frac{\partial}{\partial M^2}\phi_{\mathbf{p}}^{M^2}(\eta, \mathbf{x})\Big|_{M^2=M_\beta^2}. \quad (46)$$

The expansion of  $h_{ab}^{(S)}$  in terms of these modes reads

$$\hat{h}_{ab}^{(S)} = \sum_{k=1}^2 \int [\hat{a}_k^{(S)}(\mathbf{p})h_{\mathbf{p}ab}^{(S,k)} + \text{H.c.}] \frac{d^{n-1}p}{(2\pi)^{n-1}}, \quad (47)$$

where H.c. stands for the Hermitian conjugate of the preceding term. The commutators for the operators  $\hat{a}_k^{(S)}(\mathbf{p})$  and  $\hat{a}_l^{(S)\dagger}(\mathbf{p})$  are obtained from the commutation relations (10) with the matrix  $M$  determined through the symplectic product of the modes (8).

We now have to determine this matrix. As we have explained before, the invariant symplectic product, obtained by letting  $p^{abc} = p_{\text{inv}}^{abc}$  in Eq. (4), vanishes identically between solutions of  $L_{ab}^{(\text{inv})cd}h_{cd} = 0$  and gauge modes of the form  $h_{ab} = 2\nabla_{(a}\xi_{b)}$  for arbitrary vectors  $\xi_b$ . Since the modes  $\nabla_a\nabla_b B$  are of gauge form, and solve  $L_{ab}^{(\text{inv})cd}\nabla_c\nabla_d B = 0$  identically, they do not make a contribution to the invariant symplectic product. Hence we have

$$\begin{aligned} \langle h_{ab}^{(S)(1)}, h_{cd}^{(S)(2)} \rangle = & -i \int_\Sigma \left[ \nabla_b\nabla_c B^{(1)*} p_{\text{inv},\Psi}^{(2)abc} - \nabla_b\nabla_c B^{(2)} p_{\text{inv},\Psi}^{(1)abc*} + \nabla_b\nabla_c B^{(1)*} p_{\text{gf}}^{(2)abc} - \nabla_b\nabla_c B^{(2)} p_{\text{gf}}^{(1)abc*} \right. \\ & \left. + g_{bc}\Psi^{(1)*} p^{(2)abc} - g_{bc}\Psi^{(2)} p^{(1)abc*} \right] n_a d\Sigma, \end{aligned} \quad (48)$$

where  $p_{\text{inv},\Psi}^{(k)abc}$  is the “ $\Psi$ -part” of the invariant canonical momentum current, obtained by substituting  $h_{ab} = g_{ab}\Psi^{(k)}$  into Eq. (27). Using the field equations (37) and (38), we obtain

$$p_{\text{inv},\Psi}^{(k)abc} + p_{\text{gf}}^{(k)abc} = -\frac{n-2}{2\beta}g^{bc}\nabla^a\Psi^{(k)}, \quad (49a)$$

$$g_{bc}p^{(k)abc} = -\frac{n-2}{2}\nabla^a \left[ (n-1)H^2 B^{(k)} + \frac{n}{\beta}\Psi^{(k)} \right], \quad (49b)$$

and the symplectic product (48) between two scalar

modes reduces to

$$\begin{aligned} \langle h_{ab}^{(S)(1)}, h_{cd}^{(S)(2)} \rangle &= \frac{(n-1)(n-2)}{2} H^2 \langle B^{(1)}, \Psi^{(2)} \rangle_{\text{KG}} \\ &+ \frac{(n-1)(n-2)}{2} H^2 \langle \Psi^{(1)}, B^{(2)} \rangle_{\text{KG}} \\ &- \frac{(n-2)\lambda}{4} \langle \Psi^{(1)}, \Psi^{(2)} \rangle_{\text{KG}}, \end{aligned} \quad (50)$$

with the Klein-Gordon symplectic product

$$\begin{aligned} \langle \phi^{(1)}, \phi^{(2)} \rangle_{\text{KG}} \\ \equiv i \int_{\Sigma} \left( \phi^{(1)*} \nabla^a \phi^{(2)} - \phi^{(2)} \nabla^a \phi^{(1)*} \right) n_a d\Sigma. \end{aligned} \quad (51)$$

By taking the hypersurface  $\Sigma$  to be  $\eta = \text{const}$ , which entails  $d\Sigma = (-H\eta)^{-(n-1)} d^{n-1}x$  and  $n^a = (-H\eta)\delta_0^a$ , one can readily check that the scalar field modes (40) are normalized with respect to this symplectic product

$$\langle \phi_{\mathbf{p}}^{M^2}, \phi_{\mathbf{q}}^{M^2} \rangle_{\text{KG}} = (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}). \quad (52)$$

Then, by noticing that

$$\begin{aligned} \frac{\partial}{\partial M^2} \left[ \langle \phi_{\mathbf{p}}^{M^2}, \phi_{\mathbf{q}}^{M^2} \rangle + \langle \phi_{\mathbf{p}}^{M^2}, \phi_{\mathbf{q}}^{M^2} \rangle \right]_{M^2=M_\beta^2} \\ = \frac{\partial}{\partial M^2} \left[ \langle \phi_{\mathbf{p}}^{M^2}, \phi_{\mathbf{q}}^{M^2} \rangle \right]_{M^2=M_\beta^2} = 0, \end{aligned} \quad (53)$$

one concludes that the symplectic product of the mode solutions (45) and (46) is

$$\langle h_{\mathbf{p}ab}^{(S,k)}, h_{\mathbf{q}cd}^{(S,l)} \rangle = N_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (54)$$

where the matrix  $N_{kl}$  is given by

$$N_{kl} \equiv \frac{n-2}{4} \begin{pmatrix} 0 & 2(n-1)H^2 \\ 2(n-1)H^2 & -\lambda \end{pmatrix}. \quad (55)$$

According to formula (10), the canonical commutation relations for the scalar-sector creation and annihilation operators are given by

$$[\hat{a}_k^{(S)}(\mathbf{p}), \hat{a}_l^{(S)\dagger}(\mathbf{q})] = (N^{-1})_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (56)$$

where the inverse matrix  $(N^{-1})_{kl}$  reads

$$(N^{-1})_{kl} = \frac{1}{(n-1)^2(n-2)H^2} \begin{pmatrix} \lambda H^{-2} & 2(n-1) \\ 2(n-1) & 0 \end{pmatrix} \quad (57)$$

and all other commutators vanish. The vacuum state annihilated by the  $\hat{a}_k^{(S)}$  is the Bunch-Davies vacuum, which reduces to the usual Minkowski vacuum as  $H \rightarrow 0$ . According to the general formula (13), the scalar sector of the graviton two-point function is then given by

$$\Delta_{abc'd'}^{(S)}(x, x') = \sum_{k,l=1}^2 (N^{-1})_{kl} \int h_{\mathbf{p}ab}^{(S,k)}(x) h_{\mathbf{p}c'd'}^{(S,l)*}(x') \frac{d^{n-1}p}{(2\pi)^{n-1}}. \quad (58)$$

The mode sum (58) converges for  $\beta > 0$ , and we can write the two-point function of the scalar sector as

$$\begin{aligned} \Delta_{abc'd'}^{(S)}(x, x') &= \frac{1}{(n-1)^2(n-2)H^4} \nabla_a \nabla_b \nabla_{c'} \nabla_{d'} \left[ \lambda \Delta_{M^2}(x, x') + (n-1)H^2(2n + \lambda\beta) \frac{\partial}{\partial M^2} \Delta_{M^2}(x, x') \right]_{M^2=M_\beta^2} \\ &+ \frac{2}{(n-1)(n-2)H^2} (g_{ab} \nabla_{c'} \nabla_{d'} + g_{c'd'} \nabla_a \nabla_b) \Delta_{M_\beta^2}(x, x'), \end{aligned} \quad (59)$$

with the scalar two-point function of mass  $M$

$$\Delta_{M^2}(x, x') = \int \phi_{\mathbf{p}}^{M^2}(\eta, \mathbf{x}) \phi_{\mathbf{p}}^{M^2*}(\eta', \mathbf{x}') \frac{d^{n-1}p}{(2\pi)^{n-1}}. \quad (60)$$

Inserting the concrete form of the scalar mode functions (40), we obtain

$$\Delta_{M^2}(x, x') = \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} I_\mu(x, x'), \quad (61)$$

where  $\mu$  is defined by Eq. (42) and  $I_\mu$  is found as

$$\begin{aligned} I_\mu(x, x') &\equiv 2^{n-2} \pi^{\frac{n+2}{2}} (\eta\eta')^{\frac{n-1}{2}} \\ &\times \int H_\mu^{(1)}(-|\mathbf{p}|\eta) H_\mu^{(2)}(-|\mathbf{p}|\eta') e^{i\mathbf{p}(x-x')} \frac{d^{n-1}p}{(2\pi)^{n-1}} \end{aligned} \quad (62)$$



using that

$$e^{i\mu \frac{\pi}{2}} H_\mu^{(1)}(x) \left[ e^{i\mu \frac{\pi}{2}} H_\mu^{(1)}(x') \right]^* = H_\mu^{(1)}(x) H_\mu^{(2)}(x') \quad (63)$$

for all real  $x$  and  $x'$ , and  $\mu$  either real or purely imaginary. The integral  $I_\mu$  is well known [9], and for  $n$  spacetime dimensions it was evaluated in closed form after inserting a factor of  $e^{-\epsilon|\mathbf{p}|}$  to ensure convergence for large  $|\mathbf{p}|$  in Ref. [61]. The result only depends on the de Sitter invariant

$$Z(x, x') \equiv 1 - \frac{\mathbf{r}^2 - (\eta - \eta')^2}{2\eta\eta'}, \quad (64)$$

where  $\mathbf{r} \equiv \mathbf{x} - \mathbf{x}'$ . The result reads

$$I_\mu(x, x') = \frac{\Gamma(a_+) \Gamma(a_-)}{\Gamma(\frac{n}{2})} \times {}_2F_1 \left[ a_+, a_-; \frac{n}{2}; \frac{1+Z}{2} - i\epsilon \operatorname{sgn}(\eta - \eta') \right], \quad (65)$$

where  ${}_2F_1$  is the Gauß hypergeometric function and  $a_\pm \equiv (n-1)/2 \pm \mu$ . Note that the convergence factor supplied the necessary prescription for treating the singularity of the Gauß hypergeometric function as  $Z \rightarrow 1$  so that  $I_\mu(x, x')$  becomes a well-defined bi-distribution. We will omit this explicit prescription below to simplify the notation unless it needs to be emphasized, and by abuse of notation write also  $I_\mu(x, x') = I_\mu(Z)$ .

We obtain the scalar-field two-point function necessary for our purposes here by letting  $M^2 = M_\beta^2$ . Thus it is convenient to define a new parameter  $\mu_S$  to be equal to  $\mu$  in Eq. (42) after the substitution  $M^2 = M_\beta^2 = (n-1)\beta H^2$ ,

$$\mu_S \equiv \sqrt{\frac{(n-1)^2}{4} - (n-1)\beta}. \quad (66)$$

Furthermore, to express the two-point function (59) more explicitly, we define the set of functions

$$I_\mu^{(k)}(Z) \equiv \frac{\Gamma(a_+ + k) \Gamma(a_- + k)}{2^k \Gamma(\frac{n}{2} + k)} \times {}_2F_1 \left( a_+ + k, a_- + k; \frac{n}{2} + k; \frac{1+Z}{2} \right). \quad (67)$$

For positive integer  $k$ , this is the  $k$ -th derivative of  $I_\mu(Z)$  with respect to  $Z$ , but this function is in fact well-defined for all complex  $k$  and  $\mu$  provided that  $\frac{n-1}{2} \pm \mu + k \notin \{0, -1, -2, \dots\}$ , a fact that we will use later on. From hypergeometric identities [62], one can readily show that it satisfies the relation

$$(1 - Z^2) I_\mu^{(k+2)}(Z) - (n + 2k) Z I_\mu^{(k+1)}(Z) + \left[ \mu^2 - \frac{(n-1)^2}{4} - k(n+k-1) \right] I_\mu^{(k)}(Z) = 0, \quad (68)$$

which for  $k = 0$  is nothing but the Klein-Gordon equation. We also need an expression for the derivative of  $I_\mu^{(k)}(Z)$  with respect to the parameter  $\mu$ . Thus, we define the function

$$\tilde{I}_\mu^{(k)}(Z) \equiv -\frac{1}{2\mu} \frac{\partial}{\partial \mu} I_\mu^{(k)}(Z), \quad (69)$$

the factor  $-1/(2\mu)$  has been chosen so that

$$\tilde{I}_\mu^{(k)}(Z) = H^2 \frac{\partial}{\partial M^2} I_\mu^{(k)}(Z) \quad (70)$$

if the parameter  $\mu$  is given in terms of  $M^2$  by Eq. (42). For these functions, we also obtain a recursion relation by differentiating the relation (68) with respect to  $\mu$ , which leads to

$$(1 - Z^2) \tilde{I}_\mu^{(k+2)}(Z) - (n + 2k) Z \tilde{I}_\mu^{(k+1)}(Z) + \left[ \mu^2 - \frac{(n-1)^2}{4} - k(n+k-1) \right] \tilde{I}_\mu^{(k)}(Z) = I_\mu^{(k)}(Z). \quad (71)$$

For later use, it is convenient to have an explicit expression of the two-point function (59) in terms of de Sitter-invariant bitensors and scalar functions. A complete set of such bitensors symmetric in the index pairs  $ab$  and  $c'd'$  is given by

$$T_{abc'd'}^{(1)} \equiv g_{ab} g_{c'd'}, \quad (72a)$$

$$T_{abc'd'}^{(2)} \equiv H^{-2} (g_{ab} Z_{;c'} Z_{;d'} + g_{c'd'} Z_{;a} Z_{;b}), \quad (72b)$$

$$T_{abc'd'}^{(3)} \equiv H^{-4} Z_{;a} Z_{;b} Z_{;c'} Z_{;d'}, \quad (72c)$$

$$T_{abc'd'}^{(4)} \equiv 4H^{-4} Z_{;(a} Z_{;b)(c'} Z_{;d')}, \quad (72d)$$

$$T_{abc'd'}^{(5)} \equiv 2H^{-4} Z_{;a(c'} Z_{;d')b}, \quad (72e)$$

and since we calculate the Wightman two-point function, the derivatives in Eq. (59) can be taken as if  $I_\mu$  were just a function of  $Z$ , which would not be the case were we dealing with the Feynman propagator. The prescription for the singularity of the Gauß hypergeometric function given in Eq. (65) corresponds to the Wightman two-point function, and does not generate additional local terms upon differentiation as can also be checked explicitly.

Using the relations

$$Z_{;ab} = -H^2 Z g_{ab}, \quad (73a)$$

$$Z_{;a} Z_{;a} = H^2 (1 - Z^2), \quad (73b)$$

$$Z_{;a} Z_{;ab'} = -H^2 Z Z_{;b'}, \quad (73c)$$

$$Z_{;ab'} Z_{;ac'} = H^4 \delta_{c'}^{b'} - H^2 Z_{;b'} Z_{;c'} \quad (73d)$$

for the covariant derivatives of  $Z$  [which can be obtained, e.g., by direct calculations starting from the definition of  $Z$ , Eq. (64), in the Poincaré patch], it follows that

$$\Delta_{abc'd'}^{(S)}(x, x') = \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} \sum_{k=1}^5 T_{abc'd'}^{(k)} F^{(S,k)}(Z), \quad (74)$$

with

$$F^{(S,1)} = \frac{\lambda}{(n-1)^2(n-2)} \left( Z^2 I_S^{(2)} + Z I_S^{(1)} \right) + \frac{2n + \lambda\beta}{(n-1)(n-2)} \left( Z^2 \tilde{I}_S^{(2)} + Z \tilde{I}_S^{(1)} \right) - \frac{4}{(n-1)(n-2)} Z I_S^{(1)}, \quad (75a)$$

$$F^{(S,2)} = -\frac{\lambda}{(n-1)^2(n-2)} \left( Z I_S^{(3)} + 2 I_S^{(2)} \right) - \frac{2n + \lambda\beta}{(n-1)(n-2)} \left( Z \tilde{I}_S^{(3)} + 2 \tilde{I}_S^{(2)} \right) + \frac{2}{(n-1)(n-2)} I_S^{(2)}, \quad (75b)$$

$$F^{(S,3)} = \frac{1}{(n-1)(n-2)} \left[ \frac{\lambda}{n-1} I_S^{(4)} + (2n + \lambda\beta) \tilde{I}_S^{(4)} \right], \quad (75c)$$

$$F^{(S,4)} = \frac{1}{(n-1)(n-2)} \left[ \frac{\lambda}{n-1} I_S^{(3)} + (2n + \lambda\beta) \tilde{I}_S^{(3)} \right], \quad (75d)$$

$$F^{(S,5)} = \frac{1}{(n-1)(n-2)} \left[ \frac{\lambda}{n-1} I_S^{(2)} + (2n + \lambda\beta) \tilde{I}_S^{(2)} \right], \quad (75e)$$

where we have written  $I_S^{(k)} \equiv I_{\mu_S}^{(k)}$  and  $\tilde{I}_S^{(k)} \equiv \tilde{I}_{\mu_S}^{(k)}$ .

## B. Vector sector

We now treat the vector sector, for which the classical field is given by  $h_{ab}^{(V)} = \nabla_a v_b + \nabla_b v_a$  in terms of a divergence-free vector field  $v^a$  [see Eq. (31)]. We assume  $\alpha > 0$ . Since the vector modes are pure gauge, we have  $L_{ab}^{(inv)cd} h_{cd}^{(V)} = 0$  identically, and hence Eq. (23) reduces to

$$\nabla_{(a} [\nabla^2 + (n-1)H^2 - \alpha m^2] v_{b)} = 0, \quad (76)$$

which is equivalent to

$$[\nabla^2 + (n-1)H^2 - \alpha m^2] v^a = u^a, \quad (77)$$

for an arbitrary Killing vector  $u^a$  of the background de Sitter metric. All solutions of this equation are given by the sum of a solution of the homogeneous equation and one particular solution. The particular solution can be obtained by first noticing that Killing vectors satisfy

$$0 = \nabla^a (\nabla_a u_b + \nabla_b u_a) - \nabla_b \nabla^a u_a = [\nabla^2 + (n-1)H^2] u_b. \quad (78)$$

Then, it is clear that

$$v_a = -\frac{1}{\alpha m^2} u^a \quad (79)$$

solves the inhomogeneous equation (77). The particular solution (79), however, is a Killing vector itself and,

therefore, can be discarded as it does not contribute to the vector perturbation. Hence, the relevant modes satisfy the homogeneous part of Eq. (77), which reads

$$[\nabla^2 + (n-1)H^2 - \alpha m^2] v^a = 0 \quad (80)$$

together with the constraint  $\nabla_a v^a = 0$ . One can also show that given any solution  $h_{ab}^{(V+T)}$  of Eq. (23) satisfying  $\nabla^a \nabla^b h_{ab}^{(V+T)} = 0$  and  $h^{(V+T)} = 0$ , the divergence  $\nabla^b h_{ab}^{(V+T)}$  can be reproduced by  $h_{ab}^{(V)} = \nabla_a v_b + \nabla_b v_a$ , where  $v_a$  satisfies the homogeneous equation (80). This implies that we can write  $h_{ab}^{(V+T)} = h_{ab}^{(V)} + h_{ab}^{(T)}$  where  $h_{ab}^{(T)}$  is transverse and traceless.

From Eq. (80) one sees that  $v^a$  corresponds to a Stueckelberg vector field for the gauge parameter  $\xi \rightarrow \infty$  and the mass  $M^2 = \alpha m^2 - 2(n-1)H^2$ . The canonical quantization of the Stueckelberg field on a de Sitter background and the mode-sum construction of its two-point function in the Bunch-Davies vacuum was recently carried out in Ref. [61]. Instead of using the vector two-point function in this reference, we use the symplectic method to quantize the vector sector of the graviton two-point function and also verify the results of Ref. [61] that are relevant here.

We start the construction of a complete set of modes for Eq. (80) by decomposing the components of the vector field  $v^a$  in their irreducible parts with respect to the spatial  $O(n-1)$  symmetry. Note that we use  $\mu$  and  $\nu$  for spatial indices. The constraint  $\nabla_a v^a = 0$  can be solved as

$$v_0 = T, \quad (81a)$$

$$v_\mu = (-H\eta)^{-1} W_\mu + \frac{\partial_\mu}{\Delta} \left( \partial_\eta - \frac{n-2}{\eta} \right) T, \quad (81b)$$

where the spatial vector  $W_\mu$  is spatially divergence free, i.e.,  $\eta^{\mu\nu} \partial_\mu W_\nu = 0$ , and  $\Delta^{-1}$  is the inverse of the Laplace operator with vanishing boundary conditions at (spatial) infinity. This inverse is well-defined if the function it acts on in Eq. (81b) vanishes sufficiently fast at spatial infinity. We will assume this for the moment, and comment later on the condition for it to be justified. By substituting this decomposition into the field equation (80), one obtains the following equations of motion:

$$\left( \partial_\eta^2 - \frac{n-2}{\eta} \partial_\eta - \Delta + \frac{\alpha m^2 - nH^2}{H^2 \eta^2} \right) T = 0, \quad (82a)$$

$$\left( \partial_\eta^2 - \frac{n-2}{\eta} \partial_\eta - \Delta + \frac{\alpha m^2 - nH^2}{H^2 \eta^2} \right) W_\mu = 0. \quad (82b)$$

The divergence-free spatial vector  $W_\mu$  can readily be expressed in the momentum space in terms of the polarization vectors  $e_\mu^{(k)}(\mathbf{p})$ , with  $k \in \{1, \dots, n-2\}$ , satisfying  $p^\mu e_\mu^{(k)}(\mathbf{p}) = 0$ , which form a basis of the subspace of the momentum space orthogonal to  $\mathbf{p}$ . We normalize them by imposing the condition

$$\eta^{\mu\nu} e_\mu^{(k)}(\mathbf{p}) e_\nu^{(l)*}(\mathbf{p}) = \delta_{kl}, \quad (83)$$

where  $\delta_{kl}$  is the Kronecker delta. Since the  $e_\mu^{(k)}(\mathbf{p})$  form a basis of the space of momenta orthogonal to  $\mathbf{p}$ , they also satisfy

$$\sum_{k=1}^{n-2} e_\mu^{(k)}(\mathbf{p}) e_\nu^{(k)*}(\mathbf{p}) = \tilde{P}_{\mu\nu}, \quad (84)$$

where

$$\tilde{P}_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{p_\mu p_\nu}{\mathbf{p}^2} \quad (85)$$

is the projection operator onto the subspace orthogonal to  $\mathbf{p}$ , corresponding to the projector onto the space of spatially transverse tensors

$$P_{\mu\nu} \equiv \delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \quad (86)$$

in coordinate space.

Equations (82) have exactly the form of the Klein-Gordon equation (39) with squared mass  $M^2 = \alpha m^2 - nH^2$ . Consequently the modes defining the Bunch-Davies vacuum are just the modes (40). Hence we find

$$T_{\mathbf{p}}(\eta, \mathbf{x}) \equiv \phi_{\mathbf{p}}^{\alpha m^2 - nH^2}(\eta, \mathbf{x}), \quad (87a)$$

$$W_{\mathbf{p}\mu}^{(k)}(\eta, \mathbf{x}) \equiv e_\mu^{(k)}(\mathbf{p}) \phi_{\mathbf{p}}^{\alpha m^2 - nH^2}(\eta, \mathbf{x}). \quad (87b)$$

The modes for the vector field  $v^a$  can then be determined from the decomposition (81). The modes that are spatial scalars are obtained by substituting Eq. (87a) into Eqs. (81a) and (81b) with  $W_\mu = 0$ , and read

$$v_{\mathbf{p}0}^{(S)}(\eta, \mathbf{x}) = \phi_{\mathbf{p}}^{\alpha m^2 - nH^2}(\eta, \mathbf{x}), \quad (88a)$$

$$\begin{aligned} v_{\mathbf{p}\mu}^{(S)}(\eta, \mathbf{x}) &= \frac{\partial_\mu}{\Delta} \left( \partial_\eta - \frac{n-2}{\eta} \right) \phi_{\mathbf{p}}^{\alpha m^2 - nH^2}(\eta, \mathbf{x}) \\ &= -\frac{i p_\mu}{\mathbf{p}^2} \left( \partial_\eta - \frac{n-2}{\eta} \right) \phi_{\mathbf{p}}^{\alpha m^2 - nH^2}(\eta, \mathbf{x}). \end{aligned} \quad (88b)$$

The modes that are spatial vectors are obtained by substituting Eq. (87b) into Eq. (81b) with  $T = 0$ , and read

$$v_{\mathbf{p}0}^{(V,k)}(\eta, \mathbf{x}) = 0, \quad (89a)$$

$$v_{\mathbf{p}\mu}^{(V,k)}(\eta, \mathbf{x}) = -(H\eta)^{-1} e_\mu^{(k)}(\mathbf{p}) \phi_{\mathbf{p}}^{\alpha m^2 - nH^2}(\eta, \mathbf{x}). \quad (89b)$$

Thus, the modes for the field  $h_{ab}^{(V)}$  given by Eq. (31) are

$$h_{\mathbf{p}ab}^{(VS)} \equiv \nabla_a v_{\mathbf{p}b}^{(S)} + \nabla_b v_{\mathbf{p}a}^{(S)} \quad (90)$$

and

$$h_{\mathbf{p}ab}^{(VV,k)} \equiv \nabla_a v_{\mathbf{p}b}^{(V,k)} + \nabla_b v_{\mathbf{p}a}^{(V,k)}. \quad (91)$$

Hence, the vector sector of the quantum operator  $\hat{h}_{ab}$  can be expanded as follows:

$$\begin{aligned} \hat{h}_{ab}^{(V)} &= \int \left[ \hat{a}^{(VS)}(\mathbf{p}) h_{\mathbf{p}ab}^{(VS)} + \text{H.c.} \right] \frac{d^{n-1}p}{(2\pi)^{n-1}} \\ &+ \sum_{k=1}^{n-2} \int \left[ \hat{a}_k^{(VV)}(\mathbf{p}) h_{\mathbf{p}ab}^{(VV,k)} + \text{H.c.} \right] \frac{d^{n-1}p}{(2\pi)^{n-1}}. \end{aligned} \quad (92)$$

The commutation relations for the creation and annihilation operators in this expansion are again obtained from the symplectic product (4) of the modes (90) and (91). The symplectic product (4) for the vector sector reduces to

$$\left\langle h_{ab}^{(V)(1)}, h_{cd}^{(V)(2)} \right\rangle = -m^2 \left\langle v_a^{(1)}, v_b^{(2)} \right\rangle_V, \quad (93)$$

with the “vector” symplectic product  $\langle \cdot, \cdot \rangle_V$  given by

$$\left\langle v_a^{(1)}, v_b^{(2)} \right\rangle_V \equiv i \int_\Sigma \left( v^{(1)b*} \nabla^a v_b^{(2)} - v^{(2)b} \nabla^a v_b^{(1)*} \right) n_a d\Sigma \quad (94)$$

and conserved for solutions of Eq. (80). We then calculate (e.g., by taking  $\Sigma$  to be  $\eta = \text{const}$ ) the symplectic product (4) for the modes (90) and (91) and obtain

$$\begin{aligned} \left\langle h_{\mathbf{p}ab}^{(VS)}, h_{\mathbf{q}cd}^{(VS)} \right\rangle &= -m^2 [\alpha m^2 - 2(n-1)H^2] |\mathbf{p}|^{-2} \\ &\times (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \end{aligned} \quad (95a)$$

$$\left\langle h_{\mathbf{p}ab}^{(VV,k)}, h_{\mathbf{q}cd}^{(VV,l)} \right\rangle = -m^2 \delta_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (95b)$$

$$\left\langle h_{\mathbf{p}ab}^{(VS)}, h_{\mathbf{q}cd}^{(VV,k)} \right\rangle = 0. \quad (95c)$$

Thus, the commutation relations for the creation and annihilation operators of the vector sector are given by

$$\begin{aligned} \left[ \hat{a}^{(VS)}(\mathbf{p}), \hat{a}^{(VS)\dagger}(\mathbf{q}) \right] &= -\frac{\mathbf{p}^2}{m^2 [\alpha m^2 - 2(n-1)H^2]} \\ &\times (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \end{aligned} \quad (96a)$$

$$\left[ \hat{a}_k^{(VV)}(\mathbf{p}), \hat{a}_l^{(VV)\dagger}(\mathbf{q}) \right] = -\frac{1}{m^2} \delta_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (96b)$$

with all other commutators vanishing.

The vector sector of the graviton two-point function can then be constructed from these modes according to Eq. (13) and reads

$$\begin{aligned} \Delta_{abc'd'}^{(V,m^2)}(x, x') &= N_\alpha^{(S)} \Delta_{abc'd'}^{(VS,m^2)}(x, x') \\ &+ N^{(V)} \Delta_{abc'd'}^{(VV,m^2)}(x, x'), \end{aligned} \quad (97)$$

with the spatial scalar part

$$\begin{aligned} \Delta_{abc'd'}^{(VS,m^2)}(x, x') &= -\frac{(4\pi)^{\frac{n}{2}}}{4H^{n+2}} \int \mathbf{p}^2 h_{\mathbf{p}ab}^{(VS)}(x) h_{\mathbf{p}c'd'}^{(VS)*}(x') \frac{d^{n-1}p}{(2\pi)^{n-1}}, \end{aligned} \quad (98)$$

the spatial vector part

$$\begin{aligned} \Delta_{abc'd'}^{(VV,m^2)}(x, x') &= -\frac{(4\pi)^{\frac{n}{2}}}{H^n} \sum_{k=1}^{n-2} \int h_{\mathbf{p}ab}^{(VV,k)}(x) h_{\mathbf{p}c'd'}^{(VV,k)}(x') \frac{d^{n-1}p}{(2\pi)^{n-1}}, \end{aligned} \quad (99)$$

and the constants

$$N_\alpha^{(S)} \equiv \frac{4H^4}{m^2[\alpha m^2 - 2(n-1)H^2]} \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} \quad (100)$$

and

$$N^{(V)} \equiv \frac{H^2}{m^2} \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}}. \quad (101)$$

The explicit form of the mode functions (88) and (89) shows that the momentum integrals (98) and (99) converge in the IR if the squared mass  $M^2 = \alpha m^2 - nH^2$  of the scalar mode functions is positive, since the factor of  $p_\mu p_\nu / \mathbf{p}^4 \sim 1/\mathbf{p}^2$  gets canceled by the explicit factor of  $\mathbf{p}^2$  in the momentum integral (98). We thus assume  $M^2 > 0$ , i.e.,  $\alpha m^2 > nH^2$ , and discuss this condition in more detail in Sec. IV. Using the explicit mode functions (88), we obtain for the spatially-scalar part

$$\Delta_{abc'd'}^{(VS, m^2)}(x, x') = H^{-4} \nabla_{(c'} \nabla_{|a} K_{b)|d'}^{(S)}(x, x'), \quad (102)$$

where

$$K_{00'}^{(S)} = \Delta I_V(Z), \quad (103a)$$

$$K_{0\nu'}^{(S)} = \partial_{\nu'} \left( \partial_{\eta'} - \frac{n-2}{\eta'} \right) I_V(Z), \quad (103b)$$

$$K_{\mu 0'}^{(S)} = \partial_\mu \left( \partial_\eta - \frac{n-2}{\eta} \right) I_V(Z), \quad (103c)$$

$$K_{\mu\nu'}^{(S)} = \frac{\partial_\mu \partial_{\nu'}}{\Delta} \left( \partial_\eta - \frac{n-2}{\eta} \right) \left( \partial_{\eta'} - \frac{n-2}{\eta'} \right) I_V(Z), \quad (103d)$$

with the function  $I_V(Z) \equiv I_{\mu_V}(Z)$  given by Eq. (65). The parameter  $\mu_V$  is defined by substituting  $M^2 = \alpha m^2 - nH^2$  in Eq. (42) that gives the parameter  $\mu$  in terms of  $M^2$ . Thus,

$$\mu_V \equiv \sqrt{\frac{(n+1)^2}{4} - \frac{\alpha m^2}{H^2}}. \quad (104)$$

As for the spatially-vector part, it follows from the mode functions (89) and the polarization sum (84) that

$$\Delta_{abc'd'}^{(VV, m^2)}(x, x') = H^{-4} \nabla_{(c'} \nabla_{|a} K_{b)|d'}^{(V)}(x, x') \quad (105)$$

with

$$K_{00'}^{(V)} = K_{0\nu'}^{(V)} = K_{\mu 0'}^{(V)} = 0, \quad (106a)$$

$$K_{\mu\nu'}^{(V)} = -\frac{4}{\eta\eta'} P_{\mu\nu} I_V(Z). \quad (106b)$$

In order to obtain a covariant expression for  $\Delta_{abc'd'}^{(V, m^2)}(x, x')$  we need to further simplify the spatially-scalar contribution to the vector two-point function,  $K_{ab'}^{(S)}$ . For the temporal and mixed spatial-temporal components of  $K_{ab'}^{(S)}$  the derivatives acting on  $I_V(Z)$  in

Eqs. (103a), (103b), and (103c) can readily be found using the derivatives of  $Z$ , which is defined by Eq. (64), as

$$\partial_\eta Z = \frac{1}{\eta'} - \frac{Z}{\eta}, \quad (107a)$$

$$\partial_{\eta'} Z = \frac{1}{\eta} - \frac{Z}{\eta'}, \quad (107b)$$

$$\partial_\eta \partial_{\eta'} Z = -\frac{1}{\eta^2} - \frac{1}{(\eta')^2} + \frac{Z}{\eta\eta'}, \quad (107c)$$

and

$$\partial_\mu Z = -\frac{r_\mu}{\eta\eta'}, \quad (108a)$$

$$\partial_{\nu'} Z = \frac{r_{\nu'}}{\eta\eta'}, \quad (108b)$$

$$\partial_\mu \partial_{\nu'} Z = -\frac{\eta_{\mu\nu'}}{\eta\eta'} \quad (108c)$$

with  $r_\mu \equiv (\mathbf{x} - \mathbf{x}')_\mu$ . The calculation for the purely spatial components,  $K_{\mu\nu'}^{(S)}$ , given by Eq. (103d) is more complicated. First, note that the action of the Laplacian on the functions  $I_\mu^{(k)}$  defined by Eq. (67) can be found using the relations (108) as

$$\Delta I_\mu^{(k)} = -\frac{(n-1)}{\eta\eta'} I_\mu^{(k+1)} + \frac{\mathbf{r}^2}{\eta^2(\eta')^2} I_\mu^{(k+2)}. \quad (109)$$

Similarly, the time derivatives of  $I_\mu^{(k)}$  can be found by using the relations (107). Thus, we obtain

$$(\eta\partial_\eta + \eta'\partial_{\eta'}) I_\mu^{(k)} = \eta\eta' \Delta I_\mu^{(k-1)} + (n-1) I_\mu^{(k)} \quad (110)$$

and

$$\begin{aligned} \eta\eta' \partial_\eta \partial_{\eta'} I_\mu^{(k)} &= \left[ \frac{(n-1)^2}{4} - \mu^2 \right] I_\mu^{(k)} - \frac{\eta^2 + (\eta')^2}{2} \Delta I_\mu^{(k)} \\ &\quad + \frac{n+1}{2} \eta\eta' \Delta I_\mu^{(k-1)} + \frac{(\eta\eta')^2}{2} \Delta^2 I_\mu^{(k-2)}, \end{aligned} \quad (111)$$

where the equality (68) satisfied by the  $I_\mu^{(k)}$  has also been used. These equations allows us to trade time derivatives for Laplacians. Thus, defining

$$K_{bd'} \equiv K_{bd'}^{(S)} + \frac{N^{(V)}}{N_\alpha^{(S)}} K_{bd'}^{(V)}, \quad (112)$$

we find [using  $\partial_{\nu'} I_\mu(Z) = -\partial_\nu I_\mu(Z)$ ]

$$\begin{aligned} K_{\mu\nu'} &= -\frac{\partial_\mu \partial_\nu}{\Delta} \partial_\eta \partial_{\eta'} I_V(Z) - \frac{(n-2)^2}{\eta\eta'} \frac{\partial_\mu \partial_\nu}{\Delta} I_V(Z) \\ &\quad + \frac{n-2}{\eta\eta'} \frac{\partial_\mu \partial_\nu}{\Delta} (\eta\partial_\eta + \eta'\partial_{\eta'}) I_V(Z) \\ &\quad - \frac{\alpha m^2 - 2(n-1)H^2}{H^2 \eta\eta'} \left( \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Delta} \right) I_V(Z). \end{aligned} \quad (113)$$

Recalling that  $I_V(Z) = I_V^{(0)}(Z)$ , where  $I_V = I_{\mu\nu}$  with  $\mu\nu$  defined by Eq. (104), we can exchange time derivatives for Laplacians using Eqs. (110) and (111). All inverse Laplacians in Eq. (113) cancel out in the process. Next, by using Eq. (109) to eliminate the remaining Laplacians, we are left with

$$K_{\mu\nu'} = -\frac{\alpha m^2 - 2(n-1)H^2}{H^2\eta\eta'}\eta_{\mu\nu}I_V^{(0)}(Z) + \partial_\mu\partial_\nu\left[ZI_V^{(0)}(Z) + (n-3)I_V^{(-1)}(Z)\right]. \quad (114)$$

We evaluate the remaining spatial derivatives using Eqs. (108). The final result turns out to be de Sitter-invariant, as expected, and has the form

$$K_{bd'} = \left[-(1-Z^2)I_V^{(2)}(Z) + (n-1)ZI_V^{(1)}(Z)\right]Z_{;bd'} - \left[ZI_V^{(2)}(Z) + (n-1)I_V^{(1)}(Z)\right]Z_{;b}Z_{;d'}, \quad (115)$$

where we have made use of Eq. (68) to simplify the final expression. This result agrees with that derived in Ref. [61], but the derivation presented here is shorter.

Note that we were able to perform the exchange of derivatives only because the momentum integral was convergent thanks to the condition  $\alpha m^2 > nH^2$  and because, thus, the inverse of the Laplacian in Eqs. (103d) and (113) was well-defined. If this condition were not satisfied, the final result would have depended on how the momentum integral was regularized (e.g., by a cutoff at low momenta, or by an additional factor of  $|\mathbf{p}|^\omega$  in the integrand).

The final result for the vector sector two-point function then reads

$$\Delta_{abc'd'}^{(V,m^2)}(x,x') = N_\alpha^{(S)}\nabla_{(c'}\nabla_{|a}K_{b)|d'}). \quad (116)$$

Again, it is convenient for later use to have an explicit expression in terms of de Sitter-invariant bitensors and scalar functions. We evaluate the derivatives using the relations (73). The result is

$$\Delta_{abc'd'}^{(V,m^2)}(x,x') = N_\alpha^{(S)}\sum_{k=1}^5 T_{abc'd'}^{(k)}F^{(V,k)}(Z), \quad (117)$$

where the bitensors  $T_{abc'd'}^{(k)}$  are defined by Eq. (72) and the coefficients are given by

$$F^{(V,1)} = -ZI_V^{(2)}, \quad (118a)$$

$$F^{(V,2)} = I_V^{(3)}, \quad (118b)$$

$$F^{(V,3)} = -ZI_V^{(4)} - (n+1)I_V^{(3)}, \quad (118c)$$

$$F^{(V,4)} = -\frac{1}{4}(1-Z^2)I_V^{(4)} + \frac{n}{4}\left(ZI_V^{(3)} - I_V^{(2)}\right), \quad (118d)$$

$$F^{(V,5)} = -\frac{1}{2}(1-Z^2)I_V^{(3)} + \frac{n}{2}ZI_V^{(2)}. \quad (118e)$$

As a check, we have verified that the two-point function  $\Delta_{abc'd'}^{(V,m^2)}(x,x')$  given by Eq. (117) is traceless in each index pair using the relations (73). [This must be the case because the vector  $v^a$ , in terms of which the vector sector of the gravitational perturbation is defined by Eq. (31), is divergence free.]

### C. Tensor sector

The tensor field  $h_{ab}^{(T)}$  is transverse and traceless, from which it follows that  $G_b = 0$  [see Eq. (19)]. Consequently, only the invariant part of the field equation (23) contributes. Thus, we have

$$(\nabla^2 - 2H^2 - m^2)h_{ab}^{(T)} = 0. \quad (119)$$

In order to find a complete set of modes we proceed similarly to the case of the vector sector, decomposing the components of  $h_{ab}^{(T)}$  in their irreducible parts with respect to the spatial  $O(n-1)$  symmetry. Enforcing (covariant) transversality and tracelessness, we obtain

$$h_{00}^{(T)} = S, \quad (120a)$$

$$h_{0\mu}^{(T)} = \frac{\partial_\mu}{\Delta}\left(\partial_\eta - \frac{n-2}{\eta}\right)S - (H\eta)^{-1}V_\mu, \quad (120b)$$

$$h_{\mu\nu}^{(T)} = \frac{\partial_\mu\partial_\nu}{\Delta}S + \frac{Q_{\mu\nu}}{n-2}\left[1 - \frac{1}{\Delta}\left(\partial_\eta - \frac{n-2}{\eta}\right)^2\right]S - 2(H\eta)^{-1}\left(\partial_\eta - \frac{n-1}{\eta}\right)\frac{\partial_{(\mu}V_{\nu)}}{\Delta} + (H\eta)^{-2}H_{\mu\nu}, \quad (120c)$$

with the spatial vector  $V_\mu$  spatially divergence free,  $\eta^{\mu\nu}\partial_\mu V_\nu = 0$ , and the spatial tensor  $H_{\mu\nu}$  spatially divergence free and traceless,  $\eta^{\mu\nu}\partial_\mu H_{\nu\rho} = 0 = \eta^{\mu\nu}H_{\mu\nu}$ . In order to shorten the formulas, we have defined the spatially traceless operator

$$Q_{\mu\nu} \equiv \eta_{\mu\nu} - (n-1)\frac{\partial_\mu\partial_\nu}{\Delta}. \quad (121)$$

As in the vector case, the inverse Laplacian in the decomposition (120) is well-defined only if the momentum integrals converge. This condition is satisfied if  $m^2 > 0$  as we shall see below. Substituting the decomposition (120) back into the field equation (119), we obtain the following equations of motion:

$$\left(\partial_\eta^2 - \frac{n-2}{\eta}\partial_\eta - \Delta + \frac{m^2}{H^2\eta^2}\right)S = 0, \quad (122a)$$

$$\left(\partial_\eta^2 - \frac{n-2}{\eta}\partial_\eta - \Delta + \frac{m^2}{H^2\eta^2}\right)V_\mu = 0, \quad (122b)$$

$$\left(\partial_\eta^2 - \frac{n-2}{\eta}\partial_\eta - \Delta + \frac{m^2}{H^2\eta^2}\right)H_{\mu\nu} = 0. \quad (122c)$$

To express the solutions of these equations, we need to define a set of spatial polarization tensors  $e_{\mu\nu}^{(k)}(\mathbf{p})$  with

$k \in \{1, \dots, n(n-3)/2\}$  which are symmetric, transverse,  $p^\mu e_{\mu\nu}^{(k)}(\mathbf{p}) = 0$ , traceless,  $\eta^{\mu\nu} e_{\mu\nu}^{(k)}(\mathbf{p}) = 0$ , and form a basis of the subspace of tensors satisfying these conditions. We normalize them by imposing the conditions

$$\eta^{\mu\nu} \eta^{\rho\sigma} e_{\mu\rho}^{(k)}(\mathbf{p}) e_{\nu\sigma}^{(l)*}(\mathbf{p}) = \delta_{kl}. \quad (123)$$

They satisfy the following completeness relation since they form a basis:

$$\sum_{k=1}^{n(n-3)/2} e_{\mu\nu}^{(k)}(\mathbf{p}) e_{\rho\sigma}^{(k)*}(\mathbf{p}) = \tilde{P}_{\mu(\rho} \tilde{P}_{\sigma)\nu} - \frac{1}{n-2} \tilde{P}_{\mu\nu} \tilde{P}_{\rho\sigma}, \quad (124)$$

where  $\tilde{P}_{\mu\nu}$  is defined by Eq. (85). Again, Eqs. (122) have exactly the form of the Klein-Gordon equation (39) with mass  $M = m$ , and, thus, the mode solutions corresponding to the Bunch-Davies vacuum are given by

$$S_{\mathbf{p}}(\eta, \mathbf{x}) = \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}), \quad (125a)$$

$$V_{\mathbf{p}\mu}^{(k)}(\eta, \mathbf{x}) = e_{\mu}^{(k)}(\mathbf{p}) \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}), \quad (125b)$$

$$H_{\mathbf{p}\mu\nu}^{(k)}(\eta, \mathbf{x}) = e_{\mu\nu}^{(k)}(\mathbf{p}) \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}). \quad (125c)$$

The modes for the field  $h_{ab}^{(\text{T})}$  can then be determined from the decomposition (120). We obtain

$$h_{00}^{(\text{TT},k)}(\eta, \mathbf{x}) = h_{0\mu}^{(\text{TT},k)}(\eta, \mathbf{x}) = 0, \quad (126a)$$

$$h_{\mu\nu}^{(\text{TT},k)}(\eta, \mathbf{x}) = (H\eta)^{-2} e_{\mu\nu}^{(k)}(\mathbf{p}) \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}) \quad (126b)$$

for the spatial tensor modes by substituting Eq. (125c) into Eq. (120) with  $S = 0 = V_\mu$ , and

$$h_{00}^{(\text{TV},k)}(\eta, \mathbf{x}) = 0, \quad (127a)$$

$$h_{0\mu}^{(\text{TV},k)}(\eta, \mathbf{x}) = -(H\eta)^{-1} e_{\mu}^{(k)}(\mathbf{p}) \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}), \quad (127b)$$

$$h_{\mu\nu}^{(\text{TV},k)}(\eta, \mathbf{x}) = \frac{2i}{H\eta} \frac{p_{(\mu} e_{\nu)}^{(k)}(\mathbf{p})}{p^2} \left( \partial_\eta - \frac{n-1}{\eta} \right) \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}) \quad (127c)$$

for the spatial vector modes by substituting Eq. (125b) into Eq. (120) with  $S = 0 = H_{\mu\nu}$ . The spatial scalar modes are found by substituting Eq. (125a) into Eq. (120) with  $V_\mu = 0 = H_{\mu\nu}$ , and using Eq. (122a) to simplify the results, as follows:

$$h_{00}^{(\text{TS})}(\eta, \mathbf{x}) = \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}), \quad (128a)$$

$$h_{0\mu}^{(\text{TS})}(\eta, \mathbf{x}) = -i \frac{p_\mu}{p^2} \left( \partial_\eta - \frac{n-2}{\eta} \right) \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}), \quad (128b)$$

$$h_{\mu\nu}^{(\text{TS})}(\eta, \mathbf{x}) = \frac{p_\mu p_\nu}{p^2} \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}) - \frac{\tilde{Q}_{\mu\nu}}{\eta^2 p^2} \times \left[ \eta \partial_\eta - (n-1) + \frac{m^2}{(n-2)H^2} \right] \phi_{\mathbf{p}}^{m^2}(\eta, \mathbf{x}), \quad (128c)$$

where

$$\tilde{Q}_{\mu\nu} = \eta_{\mu\nu} - (n-1) \frac{p_\mu p_\nu}{p^2} \quad (129)$$

is the Fourier transform of  $Q_{\mu\nu}$  defined by Eq. (121).

We expand the tensor sector of the graviton field operator,  $\hat{h}_{ab}^{(\text{T})}$ , as

$$\begin{aligned} \hat{h}_{ab}^{(\text{T})} = & \int \left[ \hat{a}^{(\text{TS})}(\mathbf{p}) h_{\mathbf{p}ab}^{(\text{TS})} + \text{H.c.} \right] \frac{d^{n-1}p}{(2\pi)^{n-1}} \\ & + \sum_{k=1}^{n-2} \int \left[ \hat{a}_k^{(\text{TV})}(\mathbf{p}) h_{\mathbf{p}ab}^{(\text{TV},k)} + \text{H.c.} \right] \frac{d^{n-1}p}{(2\pi)^{n-1}} \\ & + \sum_{k=1}^{\frac{n(n-3)}{2}} \int \left[ \hat{a}_k^{(\text{TT})}(\mathbf{p}) h_{\mathbf{p}ab}^{(\text{TT},k)} + \text{H.c.} \right] \frac{d^{n-1}p}{(2\pi)^{n-1}}. \end{aligned} \quad (130)$$

The commutation relations for the creation and annihilation operators in this expansion are obtained from the symplectic product (4) of the modes (126), (127), and (128). The calculation is again facilitated by taking the hypersurfaces  $\Sigma$  to be  $\eta = \text{const}$ . Thus, we obtain

$$\langle \hat{h}_{\mathbf{p}ab}^{(\text{TT},k)}, \hat{h}_{\mathbf{q}cd}^{(\text{TT},l)} \rangle = \frac{1}{2} \delta_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (131a)$$

$$\langle \hat{h}_{\mathbf{p}ab}^{(\text{TV},k)}, \hat{h}_{\mathbf{q}cd}^{(\text{TV},l)} \rangle = \frac{m^2}{p^2} \delta_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (131b)$$

$$\begin{aligned} \langle \hat{h}_{\mathbf{p}ab}^{(\text{TS})}, \hat{h}_{\mathbf{q}cd}^{(\text{TS})} \rangle = & \frac{(n-1)m^2[m^2 - (n-2)H^2]}{2(n-2)(p^2)^2} \\ & \times (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \end{aligned} \quad (131c)$$

and all other components vanish. Note that the symplectic norm of  $h_{\mathbf{q}ab}^{(\text{TS})}$  vanishes for  $m^2 = (n-2)H^2$ . This is a consequence of a gauge symmetry for this value of mass [63]. The commutators for the creation and annihilation operators can be found by using Eq. (10) as

$$[\hat{a}_k^{(\text{TT})}(\mathbf{p}), \hat{a}_l^{(\text{TT})\dagger}(\mathbf{q})] = 2\delta_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (132a)$$

$$[\hat{a}_k^{(\text{TV})}(\mathbf{p}), \hat{a}_l^{(\text{TV})\dagger}(\mathbf{q})] = \frac{p^2}{m^2} \delta_{kl} (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \quad (132b)$$

$$\begin{aligned} [\hat{a}^{(\text{TS})}(\mathbf{p}), \hat{a}^{(\text{TS})\dagger}(\mathbf{q})] = & \frac{2(n-2)(p^2)^2}{(n-1)m^2[m^2 - (n-2)H^2]} \\ & \times (2\pi)^{n-1} \delta^{n-1}(\mathbf{p} - \mathbf{q}), \end{aligned} \quad (132c)$$

with all others vanishing. Notice that, if  $0 < m^2 < (n-2)H^2$ , then the spatial scalar sector has negative norm [21].

The tensor sector of the Wightman two-point function in the Bunch-Davies vacuum can thus be expressed as

$$\begin{aligned} \Delta_{abc'd'}^{(\text{T},m^2)}(x, x') = & N^{(\text{V})} \Delta_{abc'd'}^{(\text{TT},m^2)}(x, x') \\ & + N^{(\text{V})} \Delta_{abc'd'}^{(\text{TV},m^2)}(x, x') \\ & + N^{(\text{S})} \Delta_{abc'd'}^{(\text{TS},m^2)}(x, x'), \end{aligned} \quad (133)$$

with the mode sums

$$\begin{aligned} \Delta_{abc'd'}^{(\text{TT},m^2)}(x,x') & \equiv 2m^2 \frac{(4\pi)^{\frac{n}{2}}}{H^n} \sum_{k=1}^{\frac{n(n-3)}{2}} \int h_{\mathbf{p}ab}^{(\text{TT},k)}(x) h_{\mathbf{p}c'd'}^{(\text{TT},k)*}(x') \frac{d^{n-1}p}{(2\pi)^{n-1}}, \end{aligned} \quad (134)$$

$$\begin{aligned} \Delta_{abc'd'}^{(\text{TV},m^2)}(x,x') & \equiv \frac{(4\pi)^{\frac{n}{2}}}{H^n} \sum_{k=1}^{n-2} \int \mathbf{p}^2 h_{\mathbf{p}ab}^{(\text{TV},k)}(x) h_{\mathbf{p}c'd'}^{(\text{TV},k)*}(x') \frac{d^{n-1}p}{(2\pi)^{n-1}}, \end{aligned} \quad (135)$$

and

$$\begin{aligned} \Delta_{abc'd'}^{(\text{TS},m^2)}(x,x') & \equiv \frac{(4\pi)^{\frac{n}{2}}}{H^{n+2}} \int (\mathbf{p}^2)^2 h_{\mathbf{p}ab}^{(\text{TS})}(x) h_{\mathbf{p}c'd'}^{(\text{TS})*}(x') \frac{d^{n-1}p}{(2\pi)^{n-1}}, \end{aligned} \quad (136)$$

and the constant

$$\begin{aligned} N^{(\text{S})} & \equiv \lim_{\alpha \rightarrow 2(n-1)/(n-2)} N_{\alpha}^{(\text{S})} \\ & = \frac{2(n-2)H^4}{(n-1)m^2[m^2 - (n-2)H^2]} \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}}, \end{aligned} \quad (137)$$

with  $N_{\alpha}^{(\text{S})}$  defined by Eq. (100) and  $N^{(\text{V})}$  given by Eq. (101). The explicit form of the mode functions (126), (127), and (128) shows that the momentum integrals (134), (135), and (136) converge in the IR if the mass  $m^2$  of the scalar mode functions is positive. This is because the factor of  $p_{\mu}p_{\nu}/\mathbf{p}^4 \sim 1/\mathbf{p}^2$  in the spatial vector part is canceled by the explicit factor of  $\mathbf{p}^2$  in the momentum integral (135), and the explicit factor of  $1/(\mathbf{p}^2)^2$  in the spatial scalar part is also canceled in the momentum integral (136). Hence, we assume  $m^2 > 0$  for the time being and treat the limit  $m^2 \rightarrow 0$  in Sec. IV.

Thus, by substituting the mode functions (126), (127), and (128) into Eqs. (134), (135), and (136), respectively, we obtain the spatial tensor, spatial vector and spatial scalar part of the tensor sector of the graviton two-point function. The spatial scalar part is given by

$$\Delta_{0000}^{(\text{TS},m^2)}(x,x') = \Delta^2 I_{\text{T}}(Z), \quad (138a)$$

$$\Delta_{000\sigma'}^{(\text{TS},m^2)}(x,x') = \left( \partial_{\eta'} - \frac{n-2}{\eta'} \right) \partial_{\sigma'} \Delta I_{\text{T}}(Z), \quad (138b)$$

$$\Delta_{00\rho'\sigma'}^{(\text{TS},m^2)}(x,x') = (\eta')^{-2} \left[ \eta' \partial_{\eta'} - (n-1) + \frac{m^2}{(n-2)H^2} \right] Q_{\rho'\sigma'} \Delta I_{\text{T}}(Z) + \partial_{\rho'} \partial_{\sigma'} \Delta I_{\text{T}}(Z), \quad (138c)$$

$$\Delta_{0\nu 0\sigma'}^{(\text{TS},m^2)}(x,x') = \left( \partial_{\eta} - \frac{n-2}{\eta} \right) \left( \partial_{\eta'} - \frac{n-2}{\eta'} \right) \partial_{\nu} \partial_{\sigma'} I_{\text{T}}(Z), \quad (138d)$$

$$\Delta_{0\nu\rho'\sigma'}^{(\text{TS},m^2)}(x,x') = \left( \partial_{\eta} - \frac{n-2}{\eta} \right) \left\{ (\eta')^{-2} \left[ \eta' \partial_{\eta'} - (n-1) + \frac{m^2}{(n-2)H^2} \right] Q_{\rho'\sigma'} + \partial_{\rho'} \partial_{\sigma'} \right\} \partial_{\nu} I_{\text{T}}(Z), \quad (138e)$$

$$\begin{aligned} \Delta_{\mu\nu\rho'\sigma'}^{(\text{TS},m^2)}(x,x') & = \left\{ \frac{Q_{\mu\nu}}{\eta^2} \left[ \eta \partial_{\eta} - (n-1) + \frac{m^2}{(n-2)H^2} \right] + \partial_{\mu} \partial_{\nu} \right\} \\ & \times \left\{ \frac{Q_{\rho'\sigma'}}{(\eta')^2} \left[ \eta' \partial_{\eta'} - (n-1) + \frac{m^2}{(n-2)H^2} \right] + \partial_{\rho'} \partial_{\sigma'} \right\} I_{\text{T}}(Z), \end{aligned} \quad (138f)$$

The nonzero components of the spatial vector part are given by

$$\Delta_{00c'd'}^{(\text{TV},m^2)}(x,x') = 0, \quad (139a)$$

$$\Delta_{0\nu 0\sigma'}^{(\text{TV},m^2)}(x,x') = -\frac{1}{H^2 \eta \eta'} P_{\nu\sigma} \Delta I_{\text{T}}(Z), \quad (139b)$$

$$\Delta_{0\nu\rho'\sigma'}^{(\text{TV},m^2)}(x,x') = \frac{2}{H^2 \eta \eta'} \left( \partial_{\eta'} - \frac{n-1}{\eta'} \right) P_{\nu(\rho} \partial_{\sigma)} I_{\text{T}}(Z), \quad (139c)$$

$$\Delta_{\mu\nu\rho'\sigma'}^{(\text{TV},m^2)}(x,x') = \frac{4}{H^2 \eta \eta'} \left( \partial_{\eta} - \frac{n-1}{\eta} \right) \left( \partial_{\eta'} - \frac{n-1}{\eta'} \right) \frac{\partial_{(\mu} P_{\nu)(\rho} \partial_{\sigma)}}{\Delta} I_{\text{T}}(Z), \quad (139d)$$

and the spatial tensor part is given by

$$\Delta_{\mu\nu\rho'\sigma'}^{(\text{TT},m^2)}(x,x') = \frac{2m^2}{H^2 \eta^2 (\eta')^2} \left( P_{\mu(\rho} P_{\sigma)\nu} - \frac{1}{n-2} P_{\mu\nu} P_{\rho\sigma} \right) I_{\text{T}}(Z), \quad (140)$$

with all components with at least one temporal index vanishing. We recall that the operators  $P_{\mu\nu}$  and  $Q_{\mu\nu}$  are defined by Eqs. (86) and (121), respectively. The function  $I_T(Z) \equiv I_{\mu_T}(Z)$  appearing above is defined by Eq. (65) with the parameter  $\mu_T$  given by

$$\mu_T \equiv \sqrt{\frac{(n-1)^2}{4} - \frac{m^2}{H^2}}. \quad (141)$$

For the  $00c'd'$  and the  $0\nu 0\sigma'$  components, the time derivatives acting on  $I_T(Z)$  can be evaluated directly with the help of Eqs. (107) and (108). For the remaining components, we proceed as in the case of the vector sector, using Eqs. (109)–(111) to trade time derivatives for Laplacians. This procedure results in

$$\begin{aligned} \Delta_{0\nu\rho'\sigma'}^{(TS,m^2)} + \frac{N^{(V)}}{N^{(S)}} \Delta_{0\nu\rho'\sigma'}^{(TV,m^2)} &= (\eta')^{-1} \partial_\nu Q_{\rho\sigma} \Delta \left\{ \frac{\mathbf{r}^2}{2\eta\eta'} I_T^{(0)}(Z) + \left[ \frac{m^2}{(n-2)H^2} - (n-2) \right] I_T^{(-1)}(Z) \right\} \\ &\quad - \frac{1}{\eta\eta'} \frac{m^2 - (n-2)H^2}{(n-2)H^2} \left( \partial_{\eta'} - \frac{n-1}{\eta'} \right) [\eta_{\rho\sigma} \partial_\nu - (n-1)\eta_{\nu(\rho} \partial_{\sigma)}] I_T^{(0)}(Z) \\ &\quad + \left( \partial_\eta - \frac{n-2}{\eta} \right) \partial_\nu \partial_\rho \partial_\sigma I_T^{(0)}(Z) \end{aligned} \quad (142)$$

and

$$\begin{aligned} \Delta_{\mu\nu\rho'\sigma'}^{(TS,m^2)} + \frac{N^{(V)}}{N^{(S)}} \left[ \Delta_{\mu\nu\rho'\sigma'}^{(TV,m^2)} + \Delta_{\mu\nu\rho'\sigma'}^{(TT,m^2)} \right] &= \frac{(n-2)\tilde{m}^2(1-\tilde{m}^2)}{\eta^2(\eta')^2} [\eta_{\mu\nu}\eta_{\rho\sigma} - (n-1)\eta_{\mu(\rho}\eta_{\sigma)\nu}] I_T^{(0)}(Z) \\ &\quad + 2 \frac{\eta_{\mu\nu}\eta_{\rho\sigma}}{(n-1)\eta\eta'} \left\{ I_T^{(1)}(Z) + (1-\tilde{m}^2) [Z I_T^{(0)}(Z) + (n-2) I_T^{(-1)}(Z)] \right\} + \partial_\mu \partial_\nu \partial_\rho \partial_\sigma I_T^{(0)}(Z) \\ &\quad + \frac{(\eta')^2 + \eta^2}{\eta^2(\eta')^2} \frac{\eta_{\mu\nu}\eta_{\rho\sigma}}{(n-1)} \left\{ Z I_T^{(1)}(Z) + [\tilde{m}^2 - (n-1)] I_T^{(0)}(Z) \right\} - (\eta_{\mu\nu} \partial_\rho \partial_\sigma + \eta_{\rho\sigma} \partial_\mu \partial_\nu) \frac{1}{\eta\eta'} I_T^{(1)}(Z) \\ &\quad - 2 \frac{(1-\tilde{m}^2)}{\eta\eta'} [\eta_{\mu\nu} \partial_\rho \partial_\sigma + \eta_{\rho\sigma} \partial_\mu \partial_\nu - (n-1) \partial_{(\mu} \eta_{\nu)(\rho} \partial_{\sigma)}] [Z I_T^{(0)}(Z) + (n-2) I_T^{(-1)}(Z)] \\ &\quad + \left( \frac{\eta_{\rho\sigma} \partial_\mu \partial_\nu}{\eta^2} + \frac{\eta_{\mu\nu} \partial_\rho \partial_\sigma}{(\eta')^2} \right) \left\{ Z I_T^{(1)}(Z) + [(n-1) - \tilde{m}^2] I_T^{(0)}(Z) \right\} \\ &\quad + \frac{\eta^2 + (\eta')^2}{(n-1)\eta^2(\eta')^2} Q_{\mu\nu} Q_{\rho\sigma} \Delta \left\{ Z I_T^{(1)}(Z) + [(n-1) - \tilde{m}^2] I_T^{(0)}(Z) \right\} \\ &\quad - \frac{1}{(n-1)\eta\eta'} Q_{\mu\nu} Q_{\rho\sigma} \Delta \left\{ 2 I_T^{(1)}(Z) + [(n+1) - 2\tilde{m}^2] Z I_T^{(0)}(Z) + [(n-2)(n+1) - (3n-5)\tilde{m}^2] I_T^{(-1)}(Z) \right\}, \end{aligned} \quad (143)$$

with the abbreviation  $\tilde{m}^2 \equiv m^2/[(n-2)H^2]$ . Except for the components (143) with all indices spatial, the remaining derivatives can now be evaluated, and the result can be cast into a de Sitter-invariant form. It is most useful to present this result in terms of de Sitter-invariant bitensors and scalar coefficient functions. Thus, we find

$$\Delta_{abc'd'}^{(T,m^2)}(x, x') = -\frac{N^{(S)}}{n-2} \sum_{k=1}^5 T_{abc'd'}^{(k)} F^{(T,k)}(Z) \quad (144)$$

with the bitensor set (72) and the coefficients

$$F^{(T,1)} = (1-Z^2)^2 I_T^{(4)} - 2(n+1)(1-Z^2) Z I_T^{(3)} - (n+1)(2-nZ^2) I_T^{(2)}, \quad (145a)$$

$$F^{(T,2)} = -(1-Z^2) I_T^{(4)} + 2(n+1) Z I_T^{(3)} + n(n+1) I_T^{(2)}, \quad (145b)$$

$$F^{(T,3)} = [1 - (n-1)Z^2] I_T^{(4)} - 2(n^2-1) Z I_T^{(3)} - n(n^2-1) I_T^{(2)}, \quad (145c)$$

$$F^{(T,4)} = -\frac{n-1}{2} (1-Z^2) Z I_T^{(4)} + \frac{n(n^2-1)}{2} Z I_T^{(2)} - \frac{n+1}{2} [n-2-2(n-1)Z^2] I_T^{(3)}, \quad (145d)$$

$$F^{(T,5)} = -\frac{n-1}{2} (1-Z^2)^2 I_T^{(4)} + (n^2-1)(1-Z^2) Z I_T^{(3)} + \frac{n(n+1)}{2} [1 - (n-1)Z^2] I_T^{(2)}, \quad (145e)$$

where we have used Eq. (68) to simplify the expressions. It can readily be verified that the right-hand side of Eq. (144) is transverse (in spacetime) and traceless with the aid of Eqs. (73).

One can show that the purely spatial components (143)



agree with Eq. (144) by proving that

$$\Delta_{\mu\nu\rho'\sigma'}^{(T,m^2)}(x,x') + \frac{N^{(S)}}{n-2} \sum_{k=1}^5 T_{\mu\nu\rho'\sigma'}^{(k)} F^{(T,k)}(Z) = 0. \quad (146)$$

Since the mode sums (134), (135), and (136) are convergent for small  $p$  as long as  $m^2 > 0$  (which we have assumed above), Eq. (146) holds if

$$\Delta \left[ \Delta_{\mu\nu\rho'\sigma'}^{(T,m^2)}(x,x') + \frac{N^{(S)}}{n-2} \sum_{k=1}^5 T_{\mu\nu\rho'\sigma'}^{(k)} F^{(T,k)}(Z) \right] = 0. \quad (147)$$

This equation can readily be checked by explicit calculation.

#### D. Overall result and the massless limit

In the preceding sections we have derived the expressions for the scalar, vector, and tensor sectors of the graviton two-point function in a two-parameter family of covariant gauges in terms of mode sums. The vector and tensor sectors have been regularized by a Fierz-Pauli mass term. As already mentioned above, while the expression for the scalar sector, Eq. (58), converges for  $m = 0$  and  $\beta > 0$ , the ones obtained for the vector and tensor sectors, Eqs. (97) and (133), are only convergent for  $\alpha m^2 > nH^2$  and  $m^2 > 0$ , respectively. Our aim now is to show that despite this fact, the sum of the vector and tensor sectors (and thus the whole graviton two-point function) is finite in the limit  $m \rightarrow 0$ . We note that, although the two-point function for the divergence-free vector field  $v_a$  satisfying Eq. (80) is defined only for  $\alpha m^2 > nH^2$ , the expression for the two-point function for  $\nabla_a v_b + \nabla_b v_a$  given by Eqs. (116) and (118) is well-defined for  $0 < \alpha m^2 < 2(n-1)H^2$  as well. Since  $nH^2 < 2(n-1)H^2$  for  $n > 2$ , we can vary  $\alpha m^2$  from above  $nH^2$  to 0 continuously. (This does not mean, however, that the momentum integrals for the vector sector is convergent with  $0 < \alpha m^2 < nH^2$ . We will find in the next section that they are IR-divergent. Thus, we are analytically continuing the vector sector obtained for  $\alpha m^2 > nH^2$  to all positive values of  $m^2$ .)

Let us first analyze the behavior of  $\Delta_{abc'd'}^{(V,m^2)}$  and  $\Delta_{abc'd'}^{(T,m^2)}$  for small  $m^2$ . We begin by noting that the final result for the vector sector, Eq. (117), diverges like  $1/m^2$  for  $m \rightarrow 0$  because of the constant factor  $N_\alpha^{(S)}$  defined by Eq. (100). For this reason we expand the coefficients  $F^{(V,k)}$  to first order in  $m^2$ . Similarly, the final result for the tensor sector, Eq. (144), diverges like  $1/m^2$  for  $m \rightarrow 0$  because of the constant  $N^{(S)}$  defined by Eq. (137), so we also expand the coefficients  $F^{(T,k)}$  to first order in  $m^2$ . The coefficients  $F^{(T,k)}$  and  $F^{(V,k)}$  are given in terms of the derivatives of  $I_T(Z)$  and  $I_V^{(1)}(Z)$  with respect to  $Z$  through Eqs. (145) and (118), respectively. Using hypergeometric identities [62], we see that, as  $m \rightarrow 0$ , the

integral  $I_T(Z)$  [with  $\mu_T$  given by Eq. (141)] diverges as

$$I_T(Z) = \frac{\Gamma(n)}{\Gamma(\frac{n}{2})} \frac{H^2}{m^2} + \mathcal{O}(m^0). \quad (148)$$

The divergent term in Eq. (148), however, is constant such that all derivatives  $I_T^{(k)}(Z)$  with  $k \geq 1$  are finite in the massless limit. Similarly, as  $m \rightarrow 0$ , the integral  $I_V^{(1)}(Z)$  [with  $\mu_V$  given in Eq. (104)] diverges as

$$I_V^{(1)}(Z) = \frac{\Gamma(n+2)}{2\Gamma(\frac{n}{2}+1)} \frac{H^2}{\alpha m^2} + \mathcal{O}(m^0), \quad (149)$$

and all higher derivatives  $I_V^{(k)}(Z)$  with  $k \geq 2$  are finite. The coefficients for the vector and tensor sectors depend only on  $I_V^{(k)}(Z)$  and  $I_T^{(k)}(Z)$  with  $k \geq 2$ . Therefore it is enough to expand the integrals  $I_\mu^{(k)}$  with  $k \geq 2$  up to linear order. By noting that

$$\lim_{m \rightarrow 0} \mu_V = \frac{n+1}{2}, \quad (150a)$$

$$\lim_{m \rightarrow 0} \mu_T = \frac{n-1}{2}, \quad (150b)$$

and using the definition (69) for the  $\mu$ -derivative of  $I_\mu(Z)$ , we find the small- $m$  limit of the integrals  $I_V^{(k)}$  and  $I_T^{(k)}$  with  $k \geq 2$  as

$$I_V^{(k)} = I_{(n+1)/2}^{(k)} + \frac{\alpha m^2}{H^2} \tilde{I}_{(n+1)/2}^{(k)} + \mathcal{O}(m^4), \quad (151a)$$

$$I_T^{(k)} = I_{(n-1)/2}^{(k)} + \frac{m^2}{H^2} \tilde{I}_{(n-1)/2}^{(k)} + \mathcal{O}(m^4). \quad (151b)$$

Now we write the vector sector and tensor sector of the two-point function, Eqs. (117) and (144), respectively, for small  $m^2$  as

$$\Delta_{abc'd'}^{(V,m^2)}(x,x') = \left[ \frac{H^2}{m^2} + \frac{\alpha}{2(n-1)} \right] \Delta_{abc'd'}^{(V,\text{div})}(x,x') + \alpha \Delta_{abc'd'}^{(V,\text{fin})}(x,x') + \mathcal{O}(m^2) \quad (152)$$

and

$$\Delta_{abc'd'}^{(T,m^2)}(x,x') = \left( \frac{H^2}{m^2} + \frac{1}{n-2} \right) \Delta_{abc'd'}^{(T,\text{div})}(x,x') + \Delta_{abc'd'}^{(T,\text{fin})}(x,x') + \mathcal{O}(m^2), \quad (153)$$

where we have introduced the following  $m$ -independent and de Sitter-invariant bitensors:

$$\begin{aligned} & \Delta_{abc'd'}^{(T/V,\text{div}/\text{fin})}(x,x') \\ &= -\frac{2}{n-1} \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} \sum_{k=1}^5 T_{abc'd'}^{(k)} F^{(T/V,\text{div}/\text{fin},k)}(Z). \end{aligned} \quad (154)$$

Here the coefficients  $F^{(T/V,\text{div}/\text{fin},k)}(Z)$  defined by

$$\begin{aligned} F^{(T/V,k)}(Z) &= F^{(T/V,\text{div},k)}(Z) \\ &+ \frac{m^2}{H^2} F^{(T/V,\text{fin},k)}(Z) + \mathcal{O}(m^4). \end{aligned} \quad (155)$$

They are obtained by substituting Eqs. (151) into Eqs. (117) and (144). We first simplify the “div” coefficients,  $F^{(T/V, \text{div}, k)}(Z)$ , further with the aid of the recursion relation (68). Since the limit  $m \rightarrow 0$  exists for the  $I_V^{(k)}$  with  $k \geq 2$  and the  $I_T^{(k)}$  with  $k \geq 1$ , we can write down these recursion relations for these cases simply by letting  $m^2 = 0$ . However, we also need these relations for  $k = 1$  in the vector sector and  $k = 0$  in the tensor sector. For these cases of the recursion relation (68) we first substitute the small- $m$  expansions (148) and (149) into Eq. (68) and then take the limit  $m \rightarrow 0$ . This procedure results in

$$(1 - Z^2)I_{(n-1)/2}^{(2)} - nZI_{(n-1)/2}^{(1)} = \frac{\Gamma(n)}{\Gamma(\frac{n}{2})}, \quad (156a)$$

$$(1 - Z^2)I_{(n+1)/2}^{(3)} - (n+2)ZI_{(n+1)/2}^{(2)} = \frac{\Gamma(n+2)}{2\Gamma(\frac{n}{2}+1)}, \quad (156b)$$

which have an additional term on the right-hand side arising as the product of a divergent term proportional to  $H^2/m^2$  and the first-order term in the expansion of  $I_\mu(Z)$  in  $m^2$ . Furthermore, by using the series expansion of the hypergeometric functions [62] we obtain the relation

$$nI_{(n-1)/2}^{(1)} = (1 - Z^2)I_{(n+1)/2}^{(2)} - \frac{\Gamma(n)}{\Gamma(\frac{n}{2})}Z, \quad (157)$$

which we can use to replace all occurrences of the  $I_{(n-1)/2}^{(k)}$  with  $k \geq 1$  by the  $I_{(n+1)/2}^{(k)}$ . Thus, we find the “div” coefficients in Eq. (154) as

$$F^{(T, \text{div}, 1)} = ZI_{(n+1)/2}^{(2)}, \quad (158a)$$

$$F^{(T, \text{div}, 2)} = -I_{(n+1)/2}^{(3)}, \quad (158b)$$

$$F^{(T, \text{div}, 3)} = ZI_{(n+1)/2}^{(4)} + (n+1)I_{(n+1)/2}^{(3)}, \quad (158c)$$

$$F^{(T, \text{div}, 4)} = ZI_{(n+1)/2}^{(3)} + \frac{n+1}{2}I_{(n+1)/2}^{(2)}, \quad (158d)$$

$$F^{(T, \text{div}, 5)} = ZI_{(n+1)/2}^{(2)} + \frac{\Gamma(n+2)}{4\Gamma(\frac{n}{2}+1)}, \quad (158e)$$

and  $F^{(V, \text{div}, k)} = -F^{(T, \text{div}, k)}$ , which is obtained using Eq. (157). This last relation implies that  $\Delta_{abc'd'}^{(V, \text{div})}(x, x') + \Delta_{abc'd'}^{(T, \text{div})}(x, x') = 0$  and, consequently, that the limit

$$\begin{aligned} \Delta_{abc'd'}^{(VT)}(x, x') &\equiv \lim_{m \rightarrow 0} [\Delta_{abc'd'}^{(V, m^2)}(x, x') + \Delta_{abc'd'}^{(T, m^2)}(x, x')] \\ &= \left[ \frac{1}{n-2} - \frac{\alpha}{2(n-1)} \right] \Delta_{abc'd'}^{(T, \text{div})}(x, x') \\ &\quad + \Delta_{abc'd'}^{(T, \text{fin})}(x, x') + \alpha \Delta_{abc'd'}^{(V, \text{fin})}(x, x') \end{aligned} \quad (159)$$

is finite.

The “fin” coefficients,  $F^{(T/V, \text{fin}, k)}(Z)$ , can be computed using the recursion relation (71). For the  $\tilde{I}_V^{(k)}$  with  $k \geq 2$  and the  $\tilde{I}_T^{(k)}$  with  $k \geq 1$  we may simply substitute  $m^2 = 0$  to find these recursion relations. For the case  $k = 1$  in the vector sector and  $k = 0$  in the tensor sector, which we also need, we first show that for small  $m$

$$\left[ \mu_T^2 - \frac{(n-1)^2}{4} \right] \tilde{I}_T^{(0)} = \frac{H^2}{m^2} \frac{\Gamma(n)}{\Gamma(\frac{n}{2})} + \mathcal{O}(m^2), \quad (160a)$$

$$\left[ \mu_V^2 - \frac{(n-1)^2}{4} - n \right] \tilde{I}_V^{(1)} = \frac{H^2}{\alpha m^2} \frac{\Gamma(n+2)}{2\Gamma(\frac{n}{2}+1)} + \mathcal{O}(m^2). \quad (160b)$$

The contribution (160) present on the left-hand side of Eq. (71) exactly cancels the divergent term on the right-hand side in the  $m \rightarrow 0$  limit [given by Eqs. (148) and (149)]. Let us define

$$\begin{aligned} \Pi_{(n-1)/2}^{(0)}(Z) &\equiv \lim_{m \rightarrow 0} \left( I_T^{(0)}(Z) - \frac{\Gamma(n)}{\Gamma(\frac{n}{2})} \frac{H^2}{m^2} \right) = \lim_{m \rightarrow 0} \left( I_T^{(0)}(Z) - I_T^{(0)}(-1) \right) - \left( \psi(n-1) + \gamma + \frac{1}{n-1} \right) \frac{\Gamma(n-1)}{\Gamma(\frac{n}{2})} \\ &= \int_{-1}^Z I_{(n-1)/2}^{(1)}(Z') dZ' - \left( \psi(n-1) + \gamma + \frac{1}{n-1} \right) \frac{\Gamma(n-1)}{\Gamma(\frac{n}{2})} \end{aligned} \quad (161)$$

and

$$\begin{aligned} \Pi_{(n+1)/2}^{(1)}(Z) &\equiv \lim_{m \rightarrow 0} \left( I_V^{(1)}(Z) - \frac{\Gamma(n+2)}{2\Gamma(\frac{n}{2}+1)} \frac{H^2}{\alpha m^2} \right) = \lim_{m \rightarrow 0} \left( I_V^{(1)}(Z) - I_V^{(1)}(-1) \right) - \left( \psi(n+1) + \gamma + \frac{1}{n+1} \right) \frac{\Gamma(n+1)}{2\Gamma(\frac{n}{2}+1)} \\ &= \int_{-1}^Z I_{(n+1)/2}^{(2)}(Z') dZ' - \left( \psi(n+1) + \gamma + \frac{1}{n+1} \right) \frac{\Gamma(n+1)}{2\Gamma(\frac{n}{2}+1)}. \end{aligned} \quad (162)$$

Then the limit  $m \rightarrow 0$  of the relation (71) for  $k = 0$  in the

tensor case and  $k = 1$  in the vector case can be written

as

$$(1 - Z^2)\tilde{I}_{(n-1)/2}^{(2)} - nZ\tilde{I}_{(n-1)/2}^{(1)} = \Pi_{(n-1)/2}^{(0)}, \quad (163a)$$

$$(1 - Z^2)\tilde{I}_{(n+1)/2}^{(3)} - (n+2)Z\tilde{I}_{(n+1)/2}^{(2)} = \Pi_{(n+1)/2}^{(1)}. \quad (163b)$$

Finally, using these relations we obtain for the “fin” coefficients as follows:

$$F^{(V,\text{fin},1)} = -Z\tilde{I}_{(n+1)/2}^{(2)}, \quad (164a)$$

$$F^{(V,\text{fin},2)} = \tilde{I}_{(n+1)/2}^{(3)}, \quad (164b)$$

$$F^{(V,\text{fin},3)} = -Z\tilde{I}_{(n+1)/2}^{(4)} - (n+1)\tilde{I}_{(n+1)/2}^{(3)}, \quad (164c)$$

$$F^{(V,\text{fin},4)} = -Z\tilde{I}_{(n+1)/2}^{(3)} - \frac{n+1}{2}\tilde{I}_{(n+1)/2}^{(2)} - \frac{1}{4}I_{(n+1)/2}^{(2)}, \quad (164d)$$

$$F^{(V,\text{fin},5)} = -Z\tilde{I}_{(n+1)/2}^{(2)} - \frac{1}{2}\Pi_{(n+1)/2}^{(1)}, \quad (164e)$$

and

$$F^{(T,\text{fin},1)} = \tilde{I}_{(n-1)/2}^{(2)} - \Pi_{(n-1)/2}^{(0)} - \frac{2}{n-2}ZI_{(n-1)/2}^{(1)} - \frac{(n-1)\Gamma(n-2)}{\Gamma(\frac{n}{2})}, \quad (165a)$$

$$F^{(T,\text{fin},2)} = -Z\tilde{I}_{(n-1)/2}^{(3)} - (n+1)\tilde{I}_{(n-1)/2}^{(2)} + \frac{1}{n-2}I_{(n-1)/2}^{(2)}, \quad (165b)$$

$$F^{(T,\text{fin},3)} = \tilde{I}_{(n-1)/2}^{(4)} + (n-1)Z\tilde{I}_{(n-1)/2}^{(3)} + (n^2-1)\tilde{I}_{(n-1)/2}^{(2)} - \frac{n-1}{n-2}I_{(n-1)/2}^{(2)}, \quad (165c)$$

$$F^{(T,\text{fin},4)} = \tilde{I}_{(n-1)/2}^{(3)} + \frac{n-1}{2}\left(Z\tilde{I}_{(n-1)/2}^{(2)} + n\tilde{I}_{(n-1)/2}^{(1)}\right) + \frac{n-1}{2(n-2)}\left[ZI_{(n-1)/2}^{(2)} + (n-2)I_{(n-1)/2}^{(1)}\right], \quad (165d)$$

$$F^{(T,\text{fin},5)} = \tilde{I}_{(n-1)/2}^{(2)} + \frac{n-1}{2}\Pi_{(n-1)/2}^{(0)} + \frac{n-1}{n-2}\left(ZI_{(n-1)/2}^{(1)} + \frac{\Gamma(n)}{2\Gamma(\frac{n}{2})}\right). \quad (165e)$$

The full two-point function for the graviton in the massless limit is given by

$$\Delta_{abc'd'}(x, x') = \Delta_{abc'd'}^{(S)}(x, x') + \Delta_{abc'd'}^{(VT)}(x, x'), \quad (166)$$

with the scalar part  $\Delta_{abc'd'}^{(S)}(x, x')$  given by Eqs. (74) and (75). The right-hand side of Eq. (166) can naturally be expressed in terms of the bitensor set (72). Indeed, by combining Eqs. (159) and (154), the two-point function (166) can be given as

$$\Delta_{abc'd'}(x, x') = \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} \sum_{k=1}^5 T_{abc'd'}^{(k)} F^{(k)}(Z) \quad (167)$$

with the coefficients

$$F^{(k)}(Z) \equiv F^{(S,k)}(Z) - \frac{2}{n-1} \left[ \frac{1}{n-2} F^{(T,\text{div},k)}(Z) + F^{(T,\text{fin},k)}(Z) \right] + \frac{\alpha}{n-1} \left[ \frac{1}{n-1} F^{(T,\text{div},k)}(Z) - 2F^{(V,\text{fin},k)}(Z) \right]. \quad (168)$$

The functions  $F^{(S,k)}$ ,  $F^{(T,\text{div},k)}$ ,  $F^{(V,\text{fin},k)}$  and  $F^{(T,\text{fin},k)}$  are given by Eqs. (75), (158), (164), and (165), respectively.

Equation (167) shows that, for the two-parameter family of covariant gauges considered here, there is a de Sitter-invariant two-point function constructed by the mode-sum method which is free of IR divergences, even though it is necessary to perform analytic continuation of the vector sector, which is of pure-gauge form, in the mass parameter  $m$  from  $\alpha m^2 > nH^2$  to all positive  $m^2$ . In the case of the scalar field theory with finite mass, it is known that, if the spacetime is global de Sitter space, there is a de Sitter-invariant state [8, 9] which coincides with the Bunch-Davies vacuum in the Poincaré patch [10] and Euclidean vacuum obtain by analytic continuation from the sphere [64]. For gravitons, it is known that on the sphere (in the Euclidean approach) the family of covariant gauges employed here leads to a propagator which is both IR-finite and, after analytic continuation, de Sitter-invariant in the full de Sitter space [28]. When restricted to the Poincaré patch, it is straightforward to check that it agrees with our result (167), where the explicit expressions in four dimensions are given in Appendix A. Thus, our result gives strong evidence for the existence of an IR-finite, de Sitter-invariant Hadamard state in the full de Sitter space whose associated Wightman two-point function coincides with Eq. (167) in the Poincaré patch, just as in the massive scalar field case, as claimed in Ref. [38].

#### E. Behavior for large separations with $n = 4$

Large separations between the two points  $x$  and  $x'$  of the two-point function corresponds to large  $Z$  (64), where large timelike separations entail  $Z \rightarrow +\infty$  while large spacelike separations result in  $Z \rightarrow -\infty$ . (Note, however, that there is no geodesic connecting the two points if  $Z < -1$ .) It is known that for  $n = 4$  the graviton two-point function grows in these limits, with the exact nature of the growth depending on the gauge parameters [28] and taking the form of a linearized gauge transformation, such that two-point functions of gauge-invariant quantities do not grow [28, 37, 38, 65]. The aim of this section is to clarify the nature of that growth for our result (167).

Since the four-dimensional case is the most interesting physically, we will specialize to  $n = 4$  in this section. Some results here can also be found in Ref. [28]. We first

note that, since the function  $I_S(Z)$  is essentially the two-point function for the scalar field of mass  $\sqrt{(n-1)\beta}H$  if  $\beta > 0$  [see Eqs. (37) and (38)], the scalar-sector two-point function  $\Delta_{abc'd'}^{(S)}(x, x')$  tends to zero as  $|Z| \rightarrow \infty$  as long as  $\beta > 0$ . We work under this assumption for the rest of this section. Thus, we only need to examine the vector and tensor sectors, which are independent of  $\beta$ . We could then expand the coefficients  $F^{(k)}$  for large  $Z$ , whose explicit expressions are given in Appendix A. However, since the bitensor set (72) is not normalized and thus itself changes with distance, we need to convert it to a normalized set. Such a set can be constructed from the tangent vectors  $n_a$  and  $n_{a'}$  to the geodesic connecting  $x$  and  $x'$ , which can be expressed via

$$n_a = -\frac{Z_{;a}}{H\sqrt{1-Z^2}} \quad (169)$$

as a function of  $Z$ , and the parallel propagator  $g_{ab'}$  which reads

$$g_{ab'} = H^{-2} \left( Z_{;ab'} - \frac{Z_{;a}Z_{;b'}}{1+Z} \right). \quad (170)$$

These bivectors were first used by Allen and Jacobson in Ref. [66]. From the relations

$$n_a n^a = 1, \quad n^a g_{ab'} = -n_{b'}, \quad g_{ab'} g^{b'c} = \delta_a^c \quad (171)$$

[which can be derived, e.g., from Eqs. (73)], one finds that they are, indeed, normalized. We now introduce the set of bitensors

$$S_{abc'd'}^{(1)} \equiv g_{ab} g_{c'd'}, \quad (172a)$$

$$S_{abc'd'}^{(2)} \equiv g_{ab} n_{c'} n_{d'} + g_{c'd'} n_a n_b, \quad (172b)$$

$$S_{abc'd'}^{(3)} \equiv n_a n_b n_{c'} n_{d'}, \quad (172c)$$

$$S_{abc'd'}^{(4)} \equiv 4n_{(a} g_{b)(c'} n_{d')}, \quad (172d)$$

$$S_{abc'd'}^{(5)} \equiv 2g_{a(c'} g_{d')b}, \quad (172e)$$

and expand the two-point function (167) in this basis, obtaining

$$\Delta_{abc'd'}(x, x') = \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} \sum_{k=1}^5 S_{abc'd'}^{(k)} G^{(k)}(Z) \quad (173)$$

with the coefficients

$$G^{(1)}(Z) = F^{(1)}(Z), \quad (174a)$$

$$G^{(2)}(Z) = (1-Z^2)F^{(2)}(Z), \quad (174b)$$

$$G^{(3)}(Z) = (1-Z^2)^2 F^{(3)}(Z) + 2(1-Z)^2 F^{(5)}(Z) + 4(1-Z)(1-Z^2)F^{(4)}(Z), \quad (174c)$$

$$G^{(4)}(Z) = (1-Z^2)F^{(4)}(Z) + (1-Z)F^{(5)}(Z), \quad (174d)$$

$$G^{(5)}(Z) = F^{(5)}(Z). \quad (174e)$$

The explicit expressions for the coefficients  $G^{(\text{TV},k)}(Z)$  are given in Appendix A. By expanding them for large  $Z$ , we obtain

$$G^{(\text{TV},1)}(Z) = 2 - \frac{4}{3} \left( 1 - \frac{3}{5}\alpha \right) \left[ \frac{11}{6} + \frac{1}{Z} + \left( 1 - \frac{1}{3Z^2} \right) \ln \left( \frac{1-Z}{2} \right) \right] + \mathcal{O}(Z^{-2}) + \mathcal{O}(Z^{-3} \ln Z), \quad (175a)$$

$$G^{(\text{TV},2)}(Z) = -\frac{2}{3} + \frac{4}{3} \left( 1 - \frac{3}{5}\alpha \right) \left[ \frac{5}{6} + \frac{1}{Z} + \left( 1 - \frac{2}{Z^2} \right) \ln \left( \frac{1-Z}{2} \right) \right] + \mathcal{O}(Z^{-2}) + \mathcal{O}(Z^{-3} \ln Z), \quad (175b)$$

$$G^{(\text{TV},3)}(Z) = 4G^{(\text{TV},4)}(Z) - 4G^{(\text{TV},2)}(Z), \quad (175c)$$

$$G^{(\text{TV},4)}(Z) = \frac{3}{5}\alpha Z - \frac{11}{3} + \frac{100}{27Z} - \left( 1 - \frac{3}{5}\alpha \right) \left[ -\frac{13}{3} + \frac{46}{27Z} + \left( -2 + \frac{20}{9Z} - \frac{4}{9Z^2} \right) \ln \left( \frac{1-Z}{2} \right) \right] + \mathcal{O}(Z^{-2}) + \mathcal{O}(Z^{-3} \ln Z), \quad (175d)$$

$$G^{(\text{TV},5)}(Z) = -2G^{(\text{TV},1)}(Z) - \frac{1}{2}G^{(\text{TV},2)}(Z). \quad (175e)$$

Note that while we can remove the terms which grow logarithmically for large  $Z$  by taking  $\alpha = 5/3$ , there will be a divergence linear in  $Z$  unless  $\alpha = 0$ . As shown in Refs. [28, 65], however, this growth does not contribute to gauge-invariant observables in linearized gravity, such as the linearized Weyl tensor [31–33].

#### IV. IR ISSUES

Let us summarize our mode-sum construction of the Wightman two-point function of gravitons in the family of covariant gauges parametrized by  $\alpha$  and  $\beta$  [see Eq. (19)] in the Poincaré patch of de Sitter space. We

introduced the Fierz-Pauli mass term [see Eq. (21)] as an infrared regularization and observed that the two-point function can be expressed as the sum of three sectors: the scalar, vector and tensor sectors. We first observed that the scalar sector can directly be obtained in the massless limit with the assumption that  $\beta > 0$ . We then found that there is no IR problem in the mode-sum construction of the vector and tensor sectors if  $\alpha m^2 > nH^2$  and  $m^2 > 0$ , respectively, with the assumption  $\alpha > 0$ . Then, the vector sector was found to be well-defined for  $0 < \alpha m^2 \leq nH^2$  as well. We used this expression of the vector sector, added it to the tensor sector and took the limit  $m \rightarrow 0$ . We found that this limit was finite. In this manner we obtained the graviton two-point function, Eq. (167), which is well-defined in the IR and invariant under the symmetry of the background de Sitter space. In this section we discuss the IR issues arising in the momentum integral of the vector and tensor sectors. In particular, we find that the momentum integral for the mixed spatial-temporal components of the vector sector is IR-divergent for  $0 < \alpha m^2 < nH^2$ . We then discuss the finiteness of the limit  $m \rightarrow 0$  for the sum of the vector and tensor sectors, which was shown in Sec. IIID, in the context of the momentum integration for the purely spatial components. We note that those components are IR-finite for all positive  $m^2$  for both vector and tensor sectors.

#### A. The IR issues for $0 < \alpha m^2 < nH^2$

If  $m^2 > 0$ , all the components of the tensor sector of the Wightman two-point function converge for small momenta and, thus, are IR-finite. On the other hand, we assumed  $\alpha m^2 > nH^2$  with  $\alpha > 0$  to guarantee the convergence of the momentum integration in the IR for the vector sector. However, the derivatives in  $h_{ab}^{(V)} = \nabla_a v_b + \nabla_b v_a$  somewhat mitigate the IR behavior, and one can readily verify that the momentum integrals for most components of  $\Delta_{abc'd'}^{(V)}(x, x')$  are IR-finite as long as  $\alpha m^2 > 0$ . However, the exceptions are the mixed spatial-temporal components with indices  $0\mu 0\nu'$ , for which the momentum integral is IR-divergent if  $0 < \alpha m^2 < nH^2$ . We discuss this problem here.

We first perform the angular part of the integrals in momentum space in Eqs. (98) and (99) through the formula

$$\int f(|\mathbf{p}|) e^{i\mathbf{p}\cdot\mathbf{x}} \frac{d^{n-1}p}{(2\pi)^{n-1}} = \frac{1}{(2\pi)^{\frac{n-1}{2}} r^{\frac{n-3}{2}}} \times \int_0^\infty f(p) J_{\frac{n-3}{2}}(pr) p^{\frac{n-1}{2}} dp, \quad (176)$$

where  $f(p)$  is any function depending only on the absolute value  $p = |\mathbf{p}|$ ,  $J_\rho(z)$  is the Bessel function of order  $\rho$ , and  $r \equiv |\mathbf{x}|$ . Then the spatial scalar and spatial vector

contributions to  $\Delta_{0\mu 0\nu'}^{(V)}(x, x')$  can be written as

$$\Delta_{0\mu 0\nu'}^{(VS, m^2)}(x, x') = \int_0^\infty \tilde{\Delta}_{0\mu 0\nu'}^{(VS, m^2)}(x, x'; p) dp, \quad (177a)$$

$$\Delta_{0\mu 0\nu'}^{(VV, m^2)}(x, x') = \int_0^\infty \tilde{\Delta}_{0\mu 0\nu'}^{(VV, m^2)}(x, x'; p) dp, \quad (177b)$$

respectively, so that the vector sector of the two-point function is given as

$$\Delta_{0\mu 0\nu'}^{(V, m^2)}(x, x') = \int_0^\infty \tilde{\Delta}_{0\mu 0\nu'}^{(V, m^2)}(x, x'; p) dp, \quad (178)$$

where

$$\begin{aligned} \tilde{\Delta}_{0\mu 0\nu'}^{(V, m^2)}(x, x'; p) &\equiv N_\alpha^{(S)} \tilde{\Delta}_{0\mu 0\nu'}^{(VS, m^2)}(x, x'; p) \\ &+ N^{(V)} \tilde{\Delta}_{0\mu 0\nu'}^{(VV, m^2)}(x, x'; p). \end{aligned} \quad (179)$$

We use the expansion for small arguments of the Hankel and Bessel functions [62] to approximate the integrands of Eqs. (177) for small  $p$  as

$$\begin{aligned} \tilde{\Delta}_{0\mu 0\nu'}^{(VS, m^2)}(x, x'; p) &= -\frac{1}{2} \left( \frac{n+1}{2} - \mu_V - \frac{\alpha m^2}{2H^2} \right) \\ &\times \left\{ \left( \frac{n+1}{2} - \mu_V - \frac{\alpha m^2}{2H^2} \right) \left[ 1 + \frac{(p\eta)^2 + (p\eta')^2}{4(\mu_V - 1)} \right] \right. \\ &\quad \left. - (2\mu_V - 3) \frac{(p\eta)^2 + (p\eta')^2}{2(\mu_V - 1)} + \mathcal{O}(p^3) \right\} \\ &\times K_{\mu\nu'}^{(S)}(\eta, \eta', pr) p^{n-2-2\mu_V} \end{aligned} \quad (180)$$

and

$$\begin{aligned} \tilde{\Delta}_{0\mu 0\nu'}^{(VV, m^2)}(x, x'; p) &= -\frac{1}{2} \left( \frac{n+1}{2} - \mu_V \right) \left\{ \left( \frac{n+1}{2} - \mu_V \right) \right. \\ &\times \left[ 1 + \frac{(p\eta)^2 + (p\eta')^2}{4(\mu_V - 1)} \right] + \frac{(p\eta)^2 + (p\eta')^2}{2(\mu_V - 1)} + \mathcal{O}(p^3) \Big\} \\ &\times K_{\mu\nu'}^{(V)}(\eta, \eta', pr) p^{n-2-2\mu_V}, \end{aligned} \quad (181)$$

where the parameter  $\mu_V$  is given by Eq. (104), and we have defined the spatial bivectors

$$\begin{aligned} K_{\mu\nu'}^{(S)}(\eta, \eta', pr) &\equiv \frac{H^{1-n}}{(4\pi)^{\frac{1}{2}}} \frac{\Gamma^2(\mu_V)}{\Gamma(\frac{n+3}{2})} (H^2 \eta \eta')^{\frac{n-5}{2}} \left( \frac{4}{\eta \eta'} \right)^{\mu_V} \\ &\times \left\{ \left[ (n+1) - \frac{1}{2}(pr)^2 \right] \eta_{\mu\nu'} + (pr)^2 \frac{r_\mu r_{\nu'}}{r^2} \right\} \end{aligned} \quad (182)$$

and

$$\begin{aligned} K_{\mu\nu'}^{(V)}(\eta, \eta', pr) &\equiv \frac{H^{1-n}}{(4\pi)^{\frac{1}{2}}} \frac{\Gamma^2(\mu_V)}{\Gamma(\frac{n+3}{2})} (H^2 \eta \eta')^{\frac{n-5}{2}} \left( \frac{4}{\eta \eta'} \right)^{\mu_V} \\ &\times \left\{ \left[ (n-2)(n+1) - \frac{n}{2}(pr)^2 \right] \eta_{\mu\nu'} + (pr)^2 \frac{r_\mu r_{\nu'}}{r^2} \right\}. \end{aligned} \quad (183)$$

As  $\alpha m^2 - nH^2 \rightarrow 0^+$ , the power of  $p$  in Eqs. (180) and (181) tends to  $-1$ . This means that the  $p$  integral diverges like  $(\alpha m^2 - nH^2)^{-1}$  in this limit. However,  $\tilde{\Delta}_{0\mu 0\nu'}^{(V,m^2)}(x, x'; p)$  behaves like  $\alpha m^2 - nH^2$  in this limit, thus canceling this pole. This agrees with the fact that the vector sector given by Eqs. (116) and (118) has no singularity at  $\alpha m^2 = nH^2$ . Nevertheless, the  $p$  integral in Eq. (178) is divergent for  $0 < \alpha m^2 < nH^2$  because the power of  $p$  in Eqs. (180) and (181) is less than  $-1$ . In particular, for small  $m^2$ , to lowest order in  $p$  we find from Eqs. (180) and (181) that both expressions behave as

$$p^{n-2-2\mu_V} = p^{-3+2\frac{\alpha m^2}{(n+1)H^2} + \mathcal{O}(m^4)}. \quad (184)$$

Thus, the  $p$  integral in Eq. (178) has a power-law divergence in the IR for  $0 < \alpha m^2 < nH^2$ . It is interesting to note that the coefficient multiplying the leading divergence is proportional to  $\alpha^2 m^4$ . Thus, if one takes the  $m \rightarrow 0$  limit before the  $p$  integration, or equivalently, if one regularizes the  $p$  integration, takes the  $m \rightarrow 0$  limit and then removes the IR regularization for the  $p$  integral, the leading power-law IR singularity we encounter here will vanish. However, as we shall see in the next subsection, this strategy will not yield a finite result in the total two-point function. As for the next-to-leading order terms in Eqs. (181) and (180), they both converge for  $m^2 > 0$  but diverge as  $m \rightarrow 0$ . Nevertheless, these terms cancel each other, rendering the final expression for  $\Delta_{0\mu 0\nu'}^{(V,m^2)}(x, x')$  finite for  $m = 0$  except for the leading divergence mentioned above. It is unclear how to justify discarding this divergence and using the expression of the vector sector, Eq. (117), analytically continued in  $m^2$  from  $\alpha m^2 > nH^2$  to study the  $m \rightarrow 0$  limit, but we conjecture that the subtraction of this divergence will not affect physical observables since the vector sector of the free graviton field comprises part of the gauge sector of the theory. We also conjecture that one will obtain the result of Sec. IIIB for all  $\alpha m^2 > 0$  without encountering IR divergences if the vector sector is constructed by the mode-sum method in global de Sitter space, because the mode sum in global de Sitter space is discrete.

### B. The $m \rightarrow 0$ limit

In the previous subsection we found that the  $p$  integrals in our graviton two-point function are IR-finite for  $m^2 > 0$  if the IR divergence of the  $0\mu 0\nu'$  component of the vector sector for  $0 < \alpha m^2 < nH^2$  is dealt with by analytic continuation in  $m^2$ . Then, the  $m \rightarrow 0$  limit of our two-point function is finite because of cancellation of terms behaving like  $1/m^2$  between the vector and tensor sectors, and is given by Eqs. (167) and (168). In this subsection we investigate this cancellation by analyzing the  $p$  integration closely for the purely spatial components. We note that these components of the vector sector as well as the tensor sector are IR-finite for

$m^2 > 0$  (with  $\alpha > 0$ ). We will show how the well-known IR divergences of the two-point function in the transverse-traceless-synchronous gauge [20], which contains only physical degrees of freedom, are canceled by gauge contributions. We will also find that the  $m \rightarrow 0$  limit and the  $p$  integral do not commute and that, if the  $m \rightarrow 0$  limit is taken before the  $p$  integration, our two-point function will be IR-divergent unless  $\alpha$  takes a particular value.

We first perform the angular integrals in the expressions for their spatial tensor, spatial vector, and spatial scalar parts, Eqs. (134)–(136), respectively, with the aid of Eq. (176) and express them as follows:

$$\Delta_{\mu\nu\rho'\sigma'}^{(\text{TT},m^2)}(x, x') = \int_0^\infty \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(\text{TT},m^2)}(x, x'; p) dp, \quad (185a)$$

$$\Delta_{\mu\nu\rho'\sigma'}^{(\text{TV},m^2)}(x, x') = \int_0^\infty \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(\text{TV},m^2)}(x, x'; p) dp, \quad (185b)$$

$$\Delta_{\mu\nu\rho'\sigma'}^{(\text{TS},m^2)}(x, x') = \int_0^\infty \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(\text{TS},m^2)}(x, x'; p) dp. \quad (185c)$$

We note that the  $m \rightarrow 0$  limit of  $\Delta_{\mu\nu\rho'\sigma'}^{(\text{TT},m^2)}(x, x')$  is the two-point function in the transverse-traceless-synchronous gauge whereas the contributions  $\Delta_{\mu\nu\rho'\sigma'}^{(\text{TV},m^2)}(x, x')$  and  $\Delta_{\mu\nu\rho'\sigma'}^{(\text{TS},m^2)}(x, x')$  are of pure-gauge form in this limit. The behavior for small  $m^2$  and  $p$  of the integrands in Eqs. (185) is given by

$$\begin{aligned} \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(\text{TT},m^2)}(x, x'; p) &= -\frac{n(n-3)}{n-2} \frac{m^2}{H^2} \\ &\times K_{\mu\nu\rho'\sigma'}^{(\mu_T)}(\eta, \eta') p^{-1+\frac{2m^2}{(n-1)H^2}} [1 + \mathcal{O}(p^2)], \end{aligned} \quad (186)$$

$$\begin{aligned} \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(\text{TV},m^2)}(x, x'; p) &= -2 \left( \frac{n-1}{2} + \mu_T \right)^2 \\ &\times K_{\mu\nu\rho'\sigma'}^{(\mu_T)}(\eta, \eta') p^{-1+\frac{2m^2}{(n-1)H^2}} [1 + \mathcal{O}(p^2)], \end{aligned} \quad (187)$$

and

$$\begin{aligned} \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(\text{TS},m^2)}(x, x'; p) &= - \left[ \frac{n-1}{2} + \mu_T - \frac{m^2}{(n-2)H^2} \right]^2 \\ &\times K_{\mu\nu\rho'\sigma'}^{(\mu_T)}(\eta, \eta') p^{-1+\frac{2m^2}{(n-1)H^2}} [1 + \mathcal{O}(p^2)], \end{aligned} \quad (188)$$

where  $\mu_T$  is defined by Eq. (141), and we have defined the following traceless spatial bitensor:

$$\begin{aligned} K_{\mu\nu\rho'\sigma'}^{(\alpha)}(\eta, \eta') &\equiv \frac{H^{1-n}}{(4\pi)^{\frac{1}{2}}} \frac{\Gamma^2(\alpha)}{\Gamma(\frac{n+3}{2})} (H^2 \eta \eta')^{\frac{n-5}{2}} \left( \frac{4}{\eta \eta'} \right)^\alpha \\ &\times [\eta_{\mu\nu} \eta_{\rho'\sigma'} - (n-1) \eta_{\mu(\rho'} \eta_{\sigma')\nu}]. \end{aligned} \quad (189)$$

The purely spatial components of the tensor sector are

thus obtained by integrating the following tensor over  $p$ :

$$\begin{aligned}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(T,m^2)}(x,x';p) &\equiv N^{(V)}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(TT,m^2)}(x,x';p) \\ &+ N^{(V)}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(TV,m^2)}(x,x';p) \\ &+ N^{(S)}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(TS,m^2)}(x,x';p),\end{aligned}\quad (190)$$

where  $N^{(V)}$  and  $N^{(S)}$ , which are proportional to  $H^2/m^2$ , are given by Eqs. (101) and (137), respectively. The contribution from the spatial tensor part, which corresponds to the two-point function in the transverse-traceless-synchronous gauge, is

$$\begin{aligned}N^{(V)}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(TT,m^2)}(x,x';p) &= -\frac{n(n-3)}{n-2}\frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}}K_{\mu\nu\rho'\sigma'}^{(\frac{n-1}{2})}(\eta,\eta') \\ &\times p^{-1+\frac{2m^2}{(n-1)H^2}}[1+\mathcal{O}(p^2)],\end{aligned}\quad (191)$$

where the terms of order  $m^2$  have been neglected except in  $p^{-1+\frac{2m^2}{(n-1)H^2}}$ . By adding the spatial vector and spatial scalar contributions, both of which become pure gauge in the limit  $m \rightarrow 0$ , we find

$$\begin{aligned}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(T,m^2)}(x,x';p) &= (n-3)\frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}}K_{\mu\nu\rho'\sigma'}^{(\frac{n-1}{2})}(\eta,\eta') \\ &\times p^{-1+\frac{2m^2}{(n-1)H^2}}[1+\mathcal{O}(p^2)].\end{aligned}\quad (192)$$

Thus, in the tensor sector the gauge contribution overcompensates the one from the transverse-traceless-synchronous modes, and changes the sign of the IR divergence.

The purely spatial components of the vector sector of the graviton two-point function can be expressed as a  $p$  integral in the same fashion as for the tensor sector. By combining Eqs. (98) and (99) with Eq. (176), one can express  $\Delta_{\mu\nu\rho'\sigma'}^{(VS,m^2)}$  and  $\Delta_{\mu\nu\rho'\sigma'}^{(VV,m^2)}$  in the following form:

$$\Delta_{\mu\nu\rho'\sigma'}^{(VS,m^2)}(x,x') = \int_0^\infty \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(VS,m^2)}(x,x';p) dp, \quad (193a)$$

$$\Delta_{\mu\nu\rho'\sigma'}^{(VV,m^2)}(x,x') = \int_0^\infty \tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(VV,m^2)}(x,x';p) dp. \quad (193b)$$

If  $m^2$  and  $p$  are small, the integrands in Eqs. (193) can be approximated as follows:

$$\begin{aligned}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(VS,m^2)}(x,x';p) &= (n-1)\left[1 - \frac{1}{(n-1)^2}\frac{\alpha m^2}{H^2}\right]\eta\eta' \\ &\times K_{\mu\nu\rho'\sigma'}^{(\mu_V)}(\eta,\eta')p^{-1+\frac{2\alpha m^2}{(n+1)H^2}}[1+\mathcal{O}(p^2)]\end{aligned}\quad (194)$$

and

$$\begin{aligned}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(VV,m^2)}(x,x';p) &= 2\eta\eta' \\ &\times K_{\mu\nu\rho'\sigma'}^{(\mu_V)}(\eta,\eta')p^{-1+\frac{2\alpha m^2}{(n+1)H^2}}[1+\mathcal{O}(p^2)].\end{aligned}\quad (195)$$

The purely spatial components of the vector sector are obtained by integrating over  $p$  the tensor

$$\begin{aligned}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(V,m^2)}(x,x';p) &\equiv N_\alpha^{(S)}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(VS,m^2)}(x,x';p) \\ &+ N^{(V)}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(VV,m^2)}(x,x';p),\end{aligned}\quad (196)$$

which for small  $m^2$  and  $p$  reads

$$\begin{aligned}\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(V,m^2)}(x,x';p) &= -\alpha\frac{n-1}{n+1}(n-3)\frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}} \\ &\times K_{\mu\nu\rho'\sigma'}^{(\frac{n-1}{2})}(\eta,\eta')p^{-1+\frac{2\alpha m^2}{(n+1)H^2}}[1+\mathcal{O}(p^2)].\end{aligned}\quad (197)$$

The  $p$  dependence of  $\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(T,m^2)}(x,x';p)$  given by Eq. (192) and the one of  $\tilde{\Delta}_{\mu\nu\rho'\sigma'}^{(V,m^2)}(x,x';p)$  given by Eq. (197) are of the form  $p^{-1+cm^2}$ , where  $c$  is a positive constant. Each of these give an IR-divergent contribution proportional to  $1/(cm^2)$  when integrated over  $p$ . These IR contributions, however, cancel exactly for any  $\alpha > 0$  when they are added together, in agreement with our conclusions in Sec. III D: the total contribution diverging like  $1/m^2$  is proportional to

$$(n-3) \cdot \frac{(n-1)H^2}{2m^2} - \alpha\frac{n-1}{n+1}(n-3) \cdot \frac{(n+1)H^2}{2\alpha m^2} = 0. \quad (198)$$

It is interesting to note that the  $m \rightarrow 0$  limit and the  $p$  integration do not commute. If one takes the  $m \rightarrow 0$  limit first, then the cancellation shown in Eq. (198) does not work for all  $\alpha$  because there will be no distinction between the small- $p$  behavior of the form  $p^{-1+\frac{2m^2}{(n-1)H^2}}$  (for the tensor sector) and that of the form  $p^{-1+\frac{2\alpha m^2}{(n+1)H^2}}$  (for the vector sector) if the limit  $m \rightarrow 0$  is taken before the  $p$  integration. In this case the IR-divergent contribution to the two-point function with an IR momentum cutoff at  $p = \lambda$  will be given by

$$\begin{aligned}\Delta_{\mu\nu\rho'\sigma'}(x,x') &\approx \frac{H^{n-2}}{(4\pi)^{\frac{n}{2}}}K_{\mu\nu\rho'\sigma'}^{(\frac{n-1}{2})}(\eta,\eta') \\ &\times (n-3)\left(1 - \alpha\frac{n-1}{n+1}\right)\ln(1/\lambda).\end{aligned}\quad (199)$$

Thus, if the  $m \rightarrow 0$  limit is taken before the  $p$  integration, our two-point function is IR-divergent unless  $\alpha = (n+1)/(n-1)$ . (Recall also that the IR divergence in the vector sector for  $0 < \alpha m^2 < nH^2$  will be absent if the  $m \rightarrow 0$  limit is taken before the  $p$  integration.) Especially, it will be IR-divergent for  $\alpha = 0$ , which corresponds to the Landau/exact gauge used in Refs. [50–52], where the two-point function constructed using a different method was found to be IR-divergent in the cutoff regularization. However, the coefficient for the IR divergence we find is different from the one found in these works. In 4 dimensions, for example, the method of Ref. [52] gives (see Ref. [53])

$$\Delta_{\mu\nu\rho'\sigma'}^{\text{MTW}}(x,x') \approx -5\frac{H^2}{(4\pi)^2}K_{\mu\nu\rho'\sigma'}^{(\frac{3}{2})}(\eta,\eta')\ln(1/\lambda), \quad (200)$$

whereas the coefficient for the IR divergence for our two-point function will be 1 instead of  $-5$  if the  $m \rightarrow 0$  limit is taken before the  $p$  integration.

## V. SUMMARY AND DISCUSSION

We have, for the first time, derived the graviton two-point function by an explicit mode-sum construction in the Poincaré patch of de Sitter space, for general space-time dimension  $n$ , in a general linear covariant gauge with parameters  $\alpha$  and  $\beta$  [see Eqs. (18) and (19)]. We added a Fierz-Pauli mass term proportional to  $m^2$  to control IR divergences and found that the two-point function can be expressed as the sum of the scalar, vector and tensor sectors. Since there is no IR problem in the scalar sector if  $\beta > 0$  [28, 52], we made this assumption and found the scalar sector directly with  $m = 0$ . As for the vector sector, we found that all relevant momentum integrals are convergent only for  $\alpha m^2 > nH^2$ . We performed the mode-sum construction with this assumption and found that the resulting vector sector of the two-point function is well-defined for any value of  $m^2 > 0$  (with  $\alpha > 0$ ) if this sector is analytically continued in  $m^2$ . The tensor sector, which contains the contribution from the transverse-traceless-synchronous modes, was found to be IR-finite as long as  $m^2 > 0$ . We then added together the tensor sector and the (analytically-continued) vector sector and found that the  $m \rightarrow 0$  limit, i.e., the limit in which the IR regulator is removed, is finite. In this manner we obtained a de Sitter invariant two-parameter family of graviton two-point function in the Poincaré patch of de Sitter space by the mode-sum method. We also verified that our result in 4 dimensions agrees with the two-point function obtained by analytic continuation from the sphere [27, 28] (see also Ref. [38]). We verified that the resulting two-point function grows for large (time- or spacelike) separation but noted that this growth does not affect local gauge-invariant observables in linearized gravity as was shown in Ref. [28].

We also considered the procedure of setting  $m = 0$  from the start and regularizing the IR divergences by an IR momentum cutoff as is usually done in the literature. We found that the graviton two-point function will be IR-divergent for a general value of  $\alpha$  because the IR divergences do not cancel between the vector and tensor sector in this case. This conclusion is in agreement with the results of Refs. [50–52] obtained for the gauge parameter  $\alpha = 0$  (the Landau or exact gauge) although there is disagreement in the exact value of the IR-divergent term. However, since the IR divergence of the vector sector is proportional to  $\alpha$ , there is one special value of  $\alpha$  where these divergences cancel exactly, namely  $\alpha = (n + 1)/(n - 1)$  in  $n$  dimensions. For this value of  $\alpha$  one encounters no IR problems at all. (We note, however, that some authors object to the gauge with nonzero  $\alpha$  [67].) If  $\alpha$  is greater than this value, the IR divergences coming from the vector sector are not

large enough to cancel the ones coming from the tensor sector, while for smaller  $\alpha$  they are too large. In particular, for the Landau gauge  $\alpha = 0$  the IR-divergent contribution coming from the vector sector and the spatial vector and spatial scalar parts of the tensor sector were found to overcompensate the “physical” IR divergence coming from the spatial transverse-traceless tensor part of the tensor sector and changes the overall sign of the IR divergence.

In Ref. [23] it was explicitly shown that these divergences are of gauge form, and can be transformed away by a “large” gauge transformation, at least in non-interacting linearized gravity (see also Refs. [22, 65]). If one considers these IR divergences to be unphysical and expects them not to contribute to correlation functions of gauge-invariant observables, the change of sign in the IR divergence does not matter (and in fact, one can then use the de Sitter-invariant two-point function from the outset). On the other hand, if one is skeptical of large gauge transformations and attributes physical reality to the IR divergences, this change of sign appears to cause difficulties because, then, one would obtain different answers depending on whether one uses the covariant two-point function with a gauge-fixing term or that corresponding to just the physical, spatial transverse-traceless metric perturbations.

Although it was shown in Ref. [37] that, as far as the computation of the correlation functions corresponding to compactly supported gauge-invariant quantities goes, the covariant graviton two-point function we have derived is physically equivalent at tree level to the de Sitter-breaking ones, the same result may not be true for non-compactly supported observables. Especially, we find it plausible that a naive calculation of observables whose support extends to spatial infinity can give different results using our two-point function versus the de Sitter-breaking two-point functions of, e.g., Refs. [50–52]. Such observables must be defined carefully (e.g., by taking their support to be contained in a sphere of radius  $R$  and showing that the limit  $R \rightarrow \infty$  is well-defined), and while studies of this sort have been undertaken in both anti-de Sitter and Minkowski spacetimes (see, e.g., Refs. [68–72]), leading to the discovery of asymptotic symmetries and conserved charges at infinity, we are not aware of a similar analysis for the de Sitter case.

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## Appendix A: Formulas for $n = 4$

Here we present the explicit form of the coefficient functions in the massless two-point function (167) in dimension  $n = 4$ .

The functions  $I_{(n+1)/2}^{(k)}$  with  $k \geq 2$  present in the tensor-sector coefficients  $F^{(T, \text{div}, k)}(Z)$ , Eq. (158), can be directly evaluated for  $n = 4$  from the definition (67). For the coefficients  $F^{(T, \text{fin}, k)}(Z)$ , given by Eq. (165), we

$$\begin{aligned} \tilde{I}_{3/2}^{(1)}(Z) &= -\frac{1}{6} \lim_{\mu \rightarrow 3/2} \frac{\partial}{\partial \mu} \sum_{\ell=0}^{\infty} \frac{\Gamma(\frac{5}{2} + \mu + \ell) \Gamma(\frac{5}{2} - \mu + \ell)}{\Gamma(3 + \ell) \ell!} \left(\frac{1+Z}{2}\right)^{\ell} \\ &= -\frac{1}{6} \sum_{\ell=0}^{\infty} \frac{3\ell^2 + 12\ell + 11}{(\ell+1)(\ell+2)} \left(\frac{1+Z}{2}\right)^{\ell} = -\frac{2(1+2Z)}{3(1-Z)(1+Z)} + \frac{2(2+Z)}{3(1+Z)^2} \ln\left(\frac{1-Z}{2}\right), \end{aligned} \quad (\text{A1})$$

and the  $\tilde{I}_{3/2}^{(k)}(Z)$  with  $k > 1$  can be obtained by simple differentiation of this expression. For the vector-sector coefficients  $F^{(V, \text{fin}, k)}(Z)$ , Eq. (164), we need the functions  $I_{(n+1)/2}^{(k)}$  with  $k \geq 2$ , which can be evaluated directly from the definition (67), the function  $\Pi_{(n+1)/2}^{(1)}(Z)$ , which can also be calculated from its definition (162), and the functions  $\tilde{I}_{(n+1)/2}^{(k)}$  with  $k \geq 2$ , for which a similar calculation as in Eq. (A1) shows

$$\begin{aligned} \tilde{I}_{5/2}^{(2)}(Z) &= \frac{2(8Z^3 + 3Z^2 - 15Z - 6)}{5(1-Z)^2(1+Z)^2} \\ &\quad + \frac{2(3Z^2 + 9Z + 8)}{5(1+Z)^3} \ln\left(\frac{1-Z}{2}\right). \end{aligned} \quad (\text{A2})$$

Finally, for the scalar-sector coefficients  $F^{(S, k)}(Z)$  given by Eq. (75) the 4-dimensional limit is straightforward for general  $\beta > 0$ . However, the limit  $\beta \rightarrow 0$  exists for the

also need the functions  $I_{(n-1)/2}^{(k)}$  with  $k \geq 1$ , which can also be evaluated directly from the definition (67), the function  $\Pi_{(n-1)/2}^{(0)}(Z)$ , which can be calculated from its definition (161), and the functions  $\tilde{I}_{(n-1)/2}^{(k)}$  with  $k \geq 1$ , which were defined by Eq. (69). For the evaluation of  $\tilde{I}_{(n-1)/2}^{(k)}$  for  $n = 4$  we first write the relevant hypergeometric function as a series in 4 dimensions and then differentiate term by term since the series is absolutely convergent. This results in

coefficients as we have

$$I_S^{(k)}(Z) \rightarrow I_{3/2}^{(k)}(Z), \quad (\text{A3a})$$

$$\tilde{I}_S^{(k)}(Z) \rightarrow \tilde{I}_{3/2}^{(k)}(Z) \quad (\text{A3b})$$

for  $k \geq 1$ . Simple expressions are also found for the case  $\beta = 2/3$ , where we obtain by a similar reasoning as above

$$I_S^{(1)}(Z) = \frac{2}{(1-Z)^2}, \quad (\text{A4a})$$

$$\tilde{I}_S^{(1)}(Z) = -\frac{2}{(1-Z)^2} - \frac{2}{(1+Z)^2} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A4b})$$

and for  $k > 1$  we again simply take derivatives. The greatest simplification occurs in the limit  $\beta \rightarrow \infty$ , for which it was shown in Ref. [61] that  $I_S^{(k)}(Z)$  tends to zero exponentially fast, and the same is seen to be true for  $\tilde{I}_S^{(k)}(Z)$ .

By combining the calculations described above, in 4 dimensions the scalar-sector coefficients  $F^{(S, k)}$  are given for  $\beta = 0$  by

$$F^{(S, 1)} = -\frac{4Z(5Z^4 - 5Z^3 - 24Z^2 + 7Z + 5)}{9(1-Z)^3(1+Z)^2} - \alpha \frac{4Z}{9(1-Z)^3} + \frac{16Z}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A5a})$$

$$F^{(S, 2)} = \frac{2(7Z^5 - 7Z^4 - 104Z^3 + 8Z^2 - 71Z + 23)}{9(1-Z)^4(1+Z)^3} + \alpha \frac{4}{3(1-Z)^4} + \frac{16}{3(1+Z)^4} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A5b})$$

$$F^{(S, 3)} = \frac{4(7Z^5 - 19Z^4 - 386Z^3 + 98Z^2 - 293Z + 17)}{9(1-Z)^5(1+Z)^4} - \alpha \frac{4(5-Z)}{3(1-Z)^5} - \frac{16(5+Z)}{3(1+Z)^5} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A5c})$$

$$F^{(S, 4)} = -\frac{4(Z^4 - 5Z^3 - 69Z^2 + 17Z - 16)}{9(1-Z)^4(1+Z)^3} - \alpha \frac{4(4-Z)}{9(1-Z)^4} + \frac{16(4+Z)}{9(1+Z)^4} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A5d})$$

$$F^{(S, 5)} = \frac{2(Z^3 + 3Z^2 + 27Z - 7)}{9(1-Z)^3(1+Z)^2} - \alpha \frac{2(3-Z)}{9(1-Z)^3} - \frac{8(3+Z)}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A5e})$$

and for  $\beta = 2/3$  by

$$F^{(S,1)} = -\frac{8Z}{3(1-Z)^3} + (9-\alpha) \left[ \frac{2Z(5Z^3 - 3Z^2 + 7Z - 1)}{9(1-Z)^3(1+Z)^2} - \frac{4(1-Z)Z}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A6a})$$

$$F^{(S,2)} = \frac{4(5+Z)}{3(1-Z)^4} - (9-\alpha) \left[ \frac{8(3Z^4 - 3Z^3 + 13Z^2 - 3Z + 2)}{9(1-Z)^4(1+Z)^3} + \frac{8(2-Z)}{9(1+Z)^4} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A6b})$$

$$F^{(S,3)} = -\frac{32}{(1-Z)^5} + (9-\alpha) \left[ \frac{8(7Z^4 - 66Z^3 + 16Z^2 - 54Z + 1)}{9(1-Z)^5(1+Z)^4} + \frac{32}{3(1+Z)^5} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A6c})$$

$$F^{(S,4)} = -\frac{8}{(1-Z)^4} + (9-\alpha) \left[ \frac{8(-3+Z-11Z^2+Z^3)}{9(1-Z)^4(1+Z)^3} - \frac{8}{3(1+Z)^4} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A6d})$$

$$F^{(S,5)} = -\frac{8}{3(1-Z)^3} - (9-\alpha) \left[ \frac{16Z}{9(1-Z)^3(1+Z)^2} - \frac{8}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A6e})$$

while they vanish in the limit  $\beta \rightarrow \infty$ . For the vector- and tensor-sector coefficients  $F^{(\text{TV},k)} \equiv F^{(k)}(Z) - F^{(S,k)}(Z)$ , we obtain

$$F^{(\text{TV},1)}(Z) = -\frac{2(-27 + 89Z - 81Z^2 + 27Z^3)}{27(1-Z)^3} + (5-3\alpha) \frac{2Z(-4 + 19Z - 73Z^2 - 15Z^3 + 33Z^4)}{135(1-Z)^3(1+Z)^2} \\ - (5-3\alpha) \frac{4Z(8 + 9Z + 3Z^2)}{45(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A7a})$$

$$F^{(\text{TV},2)}(Z) = \frac{2(17 - 12Z + 3Z^2)}{9(1-Z)^4} - (5-3\alpha) \frac{2(21 - 79Z - 14Z^2 - 14Z^3 + Z^4 + 5Z^5)}{45(1-Z)^4(1+Z)^3} \\ - (5-3\alpha) \frac{4(5 + 4Z + Z^2)}{15(1+Z)^4} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A7b})$$

$$F^{(\text{TV},3)}(Z) = \frac{2(-121 + 149Z - 75Z^2 + 15Z^3)}{9(1-Z)^5} + (5-3\alpha) \frac{4(25 + 29Z + 15Z^2 + 3Z^3)}{15(1+Z)^5} \ln\left(\frac{1-Z}{2}\right) \\ - (5-3\alpha) \frac{2(-105 + 483Z + 39Z^2 + 295Z^3 + 5Z^4 - 95Z^5 - 3Z^6 + 21Z^7)}{45(1-Z)^5(1+Z)^4}, \quad (\text{A7c})$$

$$F^{(\text{TV},4)}(Z) = \frac{8(-5 + 3Z)(7 - 7Z + 3Z^2)}{27(1-Z)^4} - (5-3\alpha) \frac{2(70 - 58Z + 297Z^2 + 52Z^3 - 148Z^4 - 18Z^5 + 45Z^6)}{135(1-Z)^4(1+Z)^3} \\ - (5-3\alpha) \frac{2(40 + 55Z + 36Z^2 + 9Z^3)}{45(1+Z)^4} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A7d})$$

$$F^{(\text{TV},5)}(Z) = \frac{(-159 + 341Z - 297Z^2 + 99Z^3)}{27(1-Z)^3} - (5-3\alpha) \frac{(-63 + 221Z + 118Z^2 - 250Z^3 - 63Z^4 + 117Z^5)}{135(1-Z)^3(1+Z)^2} \\ + (5-3\alpha) \frac{2(5 + 3Z)(3 + 4Z + 3Z^2)}{45(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A7e})$$

and we see that the logarithmic terms are absent for  $\alpha = 5/3$ , which is exactly the value of  $\alpha$  (in 4 dimensions) for which the potential IR divergences in the momentum integrals cancel if one sets  $m = 0$  before the momentum integration is performed, as explained in Sec. IV B.

In order to compare the results above with those of Ref. [28] we need to switch to the Allen-Jacobson basis [66] and use the bitensor set defined by Eqs. (172). Using the relations (174) to convert the coefficients, we obtain for the scalar-sector coefficients  $G^{(S,k)}$  for  $\beta = 0$

$$G^{(S,1)}(Z) = -\frac{4Z(5Z^4 - 5Z^3 - 24Z^2 + 7Z + 5)}{9(1-Z)^3(1+Z)^2} - \alpha \frac{4Z}{9(1-Z)^3} + \frac{16Z}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A8a})$$

$$G^{(S,2)}(Z) = \frac{2(7Z^5 - 7Z^4 - 104Z^3 + 8Z^2 - 71Z + 23)}{9(1-Z)^3(1+Z)^2} + \alpha \frac{4(1+Z)}{3(1-Z)^3} + \frac{16(1-Z)}{3(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A8b})$$

$$G^{(S,3)}(Z) = \frac{8(37 - 192Z + 192Z^2 - 310Z^3 - 21Z^4 + 6Z^5)}{9(1-Z)^3(1+Z)^2} + \alpha \frac{8(-17 - 8Z + Z^2)}{9(1-Z)^3} - \frac{32(1-Z)^2}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A8c})$$

$$G^{(S,4)}(Z) = -\frac{2(5+Z)(-5+Z-23Z^2+3Z^3)}{9(1-Z)^3(1+Z)^2} + \alpha \frac{2(-11-2Z+Z^2)}{9(1-Z)^3} + \frac{8(5+Z)(1-Z)}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A8d})$$

$$G^{(S,5)}(Z) = \frac{2(Z^3 + 3Z^2 + 27Z - 7)}{9(1-Z)^3(1+Z)^2} - \alpha \frac{2(3-Z)}{9(1-Z)^3} - \frac{8(3+Z)}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A8e})$$

and for  $\beta = 2/3$

$$G^{(S,1)}(Z) = -\frac{8Z}{3(1-Z)^3} + (9-\alpha) \left[ \frac{2Z(5Z^3 - 3Z^2 + 7Z - 1)}{9(1-Z)^3(1+Z)^2} - \frac{4(1-Z)Z}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A9a})$$

$$G^{(S,2)}(Z) = \frac{4(1+Z)(5+Z)}{3(1-Z)^3} - (9-\alpha) \left[ \frac{8(3Z^4 - 3Z^3 + 13Z^2 - 3Z + 2)}{9(1-Z)^3(1+Z)^2} + \frac{8(2-Z)(1-Z)}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A9b})$$

$$G^{(S,3)}(Z) = -\frac{16(13 + 10Z + Z^2)}{3(1-Z)^3} + (9-\alpha) \left[ \frac{8(-11 - 42Z - 24Z^2 - 22Z^3 + 3Z^4)}{9(1-Z)^3(1+Z)^2} + \frac{16(1-Z)^2}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A9c})$$

$$G^{(S,4)}(Z) = -\frac{16(2+Z)}{3(1-Z)^3} + (9-\alpha) \left[ \frac{8(-3 - Z - 9Z^2 + Z^3)}{9(1-Z)^3(1+Z)^2} - \frac{16(1-Z)}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right], \quad (\text{A9d})$$

$$G^{(S,5)}(Z) = -\frac{8}{3(1-Z)^3} - (9-\alpha) \left[ \frac{16Z}{9(1-Z)^3(1+Z)^2} - \frac{8}{9(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) \right]. \quad (\text{A9e})$$

As for the vector- and tensor-sector coefficients  $G^{(\text{TV},k)}$ , we find

$$G^{(\text{TV},1)}(Z) = -\frac{2(-27 + 89Z - 81Z^2 + 27Z^3)}{27(1-Z)^3} + (5-3\alpha) \frac{2Z(-4 + 19Z - 73Z^2 - 15Z^3 + 33Z^4)}{135(1-Z)^3(1+Z)^2} - (5-3\alpha) \frac{4Z(8 + 9Z + 3Z^2)}{45(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A10a})$$

$$G^{(\text{TV},2)}(Z) = \frac{2(1+Z)(17 - 12Z + 3Z^2)}{9(1-Z)^3} - (5-3\alpha) \frac{2(21 - 79Z - 14Z^2 - 14Z^3 + Z^4 + 5Z^5)}{45(1-Z)^3(1+Z)^2} - (5-3\alpha) \frac{4(1-Z)(5 + 4Z + Z^2)}{15(1+Z)^3} \ln\left(\frac{1-Z}{2}\right), \quad (\text{A10b})$$

$$G^{(\text{TV},3)}(Z) = 4G^{(\text{TV},4)}(Z) - 4G^{(\text{TV},2)}(Z), \quad (\text{A10c})$$

$$G^{(\text{TV},4)}(Z) = \frac{-439 + 668Z - 478Z^2 + 180Z^3 - 27Z^4}{27(1-Z)^3} - (5-3\alpha) \frac{2(1-Z)(25 + 26Z + 9Z^2)}{45(1+Z)^3} \ln\left(\frac{1-Z}{2}\right) - (5-3\alpha) \frac{77 + 168Z + 491Z^2 - 264Z^3 - 109Z^4 + 144Z^5 - 27Z^6}{135(1-Z)^3(1+Z)^2}, \quad (\text{A10d})$$

$$G^{(\text{TV},5)}(Z) = -2G^{(\text{TV},1)}(Z) - \frac{1}{2}G^{(\text{TV},2)}(Z). \quad (\text{A10e})$$

The results of Ref. [28] are given by coefficients  $f^{(\text{TV},k)}$  and  $f^{(S,k)}$  multiplying certain linear combinations of the bitensor set (172). Hence, the split between the scalar and the vector and tensor sectors in the Euclidean approach used in Ref. [28] is different from ours. This is expected because each of our scalar, vector and tensor sectors satisfy the field equations separately whereas those in the Euclidean approach do not.

Converting their expression into the bitensor set (172), we obtain

$$G^{(\text{TV},1)} + G^{(S,1)} = \frac{1}{16}f^{(\text{TV},1)} - \frac{1}{2}f^{(\text{TV},2)} + \frac{1}{16}f^{(S,1)} - \frac{1}{2}f^{(S,2)} - \frac{1}{2}f^{(S,4)} + f^{(S,5)}, \quad (\text{A11a})$$

$$G^{(\text{TV},2)} + G^{(S,2)} = -\frac{1}{4}f^{(\text{TV},1)} - \frac{1}{4}f^{(S,1)} + f^{(S,4)}, \quad (\text{A11b})$$

$$G^{(\text{TV},3)} + G^{(S,3)} = f^{(\text{TV},1)} + 4f^{(\text{TV},3)} + f^{(S,1)} + 4f^{(S,3)}, \quad (\text{A11c})$$

$$G^{(\text{TV},4)} + G^{(S,4)} = f^{(\text{TV},3)} + f^{(S,3)}, \quad (\text{A11d})$$

$$G^{(\text{TV},5)} + G^{(S,5)} = f^{(\text{TV},2)} + f^{(S,2)}, \quad (\text{A11e})$$

and using that their  $z$  is related to  $Z$  by  $z = (1+Z)/2$ , we find full agreement for the case  $\beta = 2/3$  where explicit expressions were presented. Furthermore, since only the scalar-sector two-point function depends on  $\beta$ , one readily finds from Eq. (59) that there is also agreement for all other values of  $\beta > 0$ .

- [2] G. Hinshaw *et al.* (WMAP), “Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: temperature analysis,” *Astrophys. J. Suppl.* **170**, 288 (2007), arXiv:astro-ph/0603451 [astro-ph].
- [3] A. H. Guth, “The Inflationary Universe: a possible solution to the horizon and flatness problems,” *Phys. Rev. D* **23**, 347 (1981).
- [4] A. D. Linde, “A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy, and primordial monopole problems,” *Phys. Lett. B* **108**, 389 (1982).
- [5] A. Albrecht and P. J. Steinhardt, “Cosmology for grand unified theories with radiatively induced symmetry breaking,” *Phys. Rev. Lett.* **48**, 1220 (1982).
- [6] A. G. Riess *et al.* (Supernova Search Team), “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” *Astron. J.* **116**, 1009 (1998), arXiv:astro-ph/9805201 [astro-ph].
- [7] S. Perlmutter *et al.* (Supernova Cosmology Project), “Measurements of Omega and Lambda from 42 high redshift supernovae,” *Astrophys. J.* **517**, 565 (1999), arXiv:astro-ph/9812133 [astro-ph].
- [8] N. A. Chernikov and E. A. Tagirov, “Quantum theory of scalar fields in de Sitter space-time,” *Annales H. Poincaré* **9**, 109 (1968).
- [9] C. Schomblond and P. Spindel, “Conditions d’unicité pour le propagateur  $\Delta^1(x, y)$  du champ scalaire dans l’univers de de Sitter,” *Annales H. Poincaré* **25**, 67 (1976).
- [10] T. S. Bunch and P. C. W. Davies, “Quantum field theory in de Sitter space: renormalization by point-splitting,” *Proc. Roy. Soc. Lond. A* **360**, 117 (1978).
- [11] B. Allen, “Vacuum states in de Sitter space,” *Phys. Rev. D* **32**, 3136 (1985).
- [12] L. H. Ford and L. Parker, “Infrared divergences in a class of Robertson-Walker universes,” *Phys. Rev. D* **16**, 245 (1977).
- [13] M. A. Naïmark, “Linear representations of the lorentz group,” *Transl., Ser. 2, Am. Math. Soc.* **6**, 379 (1957).
- [14] L. H. Thomas, “On unitary representations of the group of de sitter space,” *Annals Maths.* **42**, 113 (1941).
- [15] J. Dixmier, “Représentations intégrables du groupe de de sitter,” *Bull. Soc. Math. Fr.* **89**, 9 (1961).
- [16] J. G. Kuriyan, N. Mukunda, and G. Sudarshan, “Master analytic representations and unified representation theory of certain orthogonal and pseudo-orthogonal groups,” *Commun. Math. Phys.* **8**, 204 (1968).
- [17] U. Ottoson, “A classification of the unitary irreducible representations of  $SO_0(n, 1)$ ,” *Comm. Math. Phys.* **8**, 228 (1968).
- [18] A. Higuchi, “Symmetric Tensor Spherical Harmonics on the  $N$  Sphere and Their Application to the De Sitter Group  $SO(N, 1)$ ,” *J. Math. Phys.* **28**, 1553 (1987), [Erratum: *J. Math. Phys.* **43**, 6385 (2002)].
- [19] K. Kirsten and J. Garriga, “Massless minimally coupled fields in de Sitter space:  $O(4)$  symmetric states versus de Sitter invariant vacuum,” *Phys. Rev. D* **48**, 567 (1993), arXiv:gr-qc/9305013 [gr-qc].
- [20] L. H. Ford and L. Parker, “Quantized gravitational wave perturbations in Robertson-Walker universes,” *Phys. Rev. D* **16**, 1601 (1977).
- [21] A. Higuchi, “Forbidden mass range for spin-2 field theory in de Sitter space-time,” *Nucl. Phys. B* **282**, 397 (1987).
- [22] B. Allen, “The graviton propagator in homogeneous and isotropic space-times,” *Nucl. Phys. B* **287**, 743 (1987).
- [23] A. Higuchi, D. Marolf, and I. A. Morrison, “de Sitter invariance of the dS graviton vacuum,” *Class. Quant. Grav.* **28**, 245012 (2011), arXiv:1107.2712 [hep-th].
- [24] B. Allen, “The graviton propagator in de Sitter space,” *Phys. Rev. D* **34**, 3670 (1986).
- [25] B. Allen and M. Turyn, “An evaluation of the graviton propagator in de Sitter space,” *Nucl. Phys. B* **292**, 813 (1987).
- [26] S. W. Hawking, T. Hertog, and N. Turok, “Gravitational waves in open de Sitter space,” *Phys. Rev. D* **62**, 063502 (2000), arXiv:hep-th/0003016 [hep-th].
- [27] A. Higuchi and S. S. Kouris, “On the scalar sector of the covariant graviton two point function in de Sitter space-time,” *Class. Quant. Grav.* **18**, 2933 (2001), arXiv:gr-qc/0011062 [gr-qc].
- [28] A. Higuchi and S. S. Kouris, “The covariant graviton propagator in de Sitter space-time,” *Class. Quant. Grav.* **18**, 4317 (2001), arXiv:gr-qc/0107036 [gr-qc].
- [29] A. Higuchi and R. H. Weeks, “The physical graviton two point function in de Sitter space-time with  $S^3$  spatial sections,” *Class. Quant. Grav.* **20**, 3005 (2003), arXiv:gr-qc/0212031 [gr-qc].
- [30] R. P. Bernar, L. C. B. Crispino, and A. Higuchi, “Infrared-finite graviton two-point function in static de Sitter space,” *Phys. Rev. D* **90**, 024045 (2014), arXiv:1405.3827 [gr-qc].
- [31] P. J. Mora, N. C. Tsamis, and R. P. Woodard, “Weyl-Weyl correlator in de Donder gauge on de Sitter,” *Phys. Rev. D* **86**, 084016 (2012), arXiv:1205.4466 [gr-qc].
- [32] M. B. Fröb, A. Roura, and E. Verdaguer, “Riemann correlator in de Sitter including loop corrections from conformal fields,” *JCAP* **1407**, 048 (2014), arXiv:1403.3335 [gr-qc].
- [33] S. S. Kouris, “The Weyl tensor two-point function in de Sitter space-time,” *Class. Quant. Grav.* **18**, 4961 (2001), [Erratum: *Class. Quant. Grav.* **29**, 169501 (2012)], arXiv:gr-qc/0107064 [gr-qc].
- [34] M. B. Fröb, “The Weyl tensor correlator in cosmological spacetimes,” *JCAP* **1412**, 010 (2014), arXiv:1409.7964 [hep-th].
- [35] M. Faizal and A. Higuchi, “Physical equivalence between the covariant and physical graviton two-point functions in de Sitter spacetime,” *Phys. Rev. D* **85**, 124021 (2012), arXiv:1107.0395 [gr-qc].
- [36] C. J. Fewster and D. S. Hunt, “Quantization of linearized gravity in cosmological vacuum spacetimes,” *Rev. Math. Phys.* **25**, 1330003 (2013), arXiv:1203.0261 [math-ph].
- [37] A. Higuchi, “Equivalence between the Weyl-tensor and gauge-invariant graviton two-point functions in Minkowski and de Sitter spaces,” (2012), arXiv:1204.1684 [gr-qc].
- [38] I. A. Morrison, “On cosmic hair and ‘de Sitter breaking’ in linearized quantum gravity,” (2013), arXiv:1302.1860 [gr-qc].
- [39] N. C. Tsamis and R. P. Woodard, “Relaxing the cosmological constant,” *Phys. Lett. B* **301**, 351–357 (1993).
- [40] N. C. Tsamis and R. P. Woodard, “Strong infrared effects in quantum gravity,” *Annals Phys.* **238**, 1–82 (1995).
- [41] N. C. Tsamis and R. P. Woodard, “A simplified quantum gravitational model of inflation,” *Class. Quant. Grav.* **26**, 105006 (2009), arXiv:0807.5006 [gr-qc].
- [42] J. Garriga and T. Tanaka, “Can infrared gravitons screen

- $\Lambda$ ?” Phys. Rev. D **77**, 024021 (2008), arXiv:0706.0295 [hep-th].
- [43] N. C. Tsamis and R. P. Woodard, “Comment on ‘Can infrared gravitons screen  $\Lambda$ ?’,” Phys. Rev. D **78**, 028501 (2008), arXiv:0708.2004 [hep-th].
- [44] S. B. Giddings and M. S. Sloth, “Semiclassical relations and IR effects in de Sitter and slow-roll space-times,” JCAP **1101**, 023 (2011), arXiv:1005.1056 [hep-th].
- [45] S. B. Giddings and M. S. Sloth, “Cosmological observables, IR growth of fluctuations, and scale-dependent anisotropies,” Phys. Rev. D **84**, 063528 (2011), arXiv:1104.0002 [hep-th].
- [46] S. B. Giddings and M. S. Sloth, “Fluctuating geometries,  $q$ -observables, and infrared growth in inflationary space-times,” Phys. Rev. D **86**, 083538 (2012), arXiv:1109.1000 [hep-th].
- [47] H. Kitamoto and Y. Kitazawa, “Soft Gravitons Screen Couplings in de Sitter Space,” Phys. Rev. D **87**, 124007 (2013), arXiv:1203.0391 [hep-th].
- [48] H. Kitamoto and Y. Kitazawa, “Soft Graviton effects on Gauge theories in de Sitter Space,” Phys. Rev. D **87**, 124004 (2013), arXiv:1204.2876 [hep-th].
- [49] R. P. Woodard, “Perturbative quantum gravity comes of age,” Int. J. Mod. Phys. **D23**, 1430020 (2014), arXiv:1407.4748 [gr-qc].
- [50] S. P. Miao, N. C. Tsamis, and R. P. Woodard, “The graviton propagator in de Donder gauge on de Sitter background,” J. Math. Phys. **52**, 122301 (2011), arXiv:1106.0925 [gr-qc].
- [51] E. O. Kahya, S. P. Miao, and R. P. Woodard, “The coincidence limit of the graviton propagator in de Donder gauge on de Sitter background,” J. Math. Phys. **53**, 022304 (2012), arXiv:1112.4420 [gr-qc].
- [52] P. J. Mora, N. C. Tsamis, and R. P. Woodard, “Graviton propagator in a general invariant gauge on de Sitter,” J. Math. Phys. **53**, 122502 (2012), arXiv:1205.4468 [gr-qc].
- [53] S. P. Miao, P. J. Mora, N. C. Tsamis, and R. P. Woodard, “Perils of analytic continuation,” Phys. Rev. D **89**, 104004 (2014), arXiv:1306.5410 [gr-qc].
- [54] M. Fierz and W. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field,” Proc. Roy. Soc. Lond. A **173**, 211–232 (1939).
- [55] H. van Dam and M. J. G. Veltman, “Massive and massless Yang-Mills and gravitational fields,” Nucl. Phys. B **22**, 397 (1970).
- [56] V. I. Zakharov, “Linearized gravitation theory and the graviton mass,” JETP Lett. **12**, 312 (1970), [Pisma Zh. Eksp. Teor. Fiz.12,447(1970)].
- [57] A. Higuchi, “Massive symmetric tensor field in space-times with a positive cosmological constant,” Nucl. Phys. B **325**, 745 (1989).
- [58] See Ref. [73] for a formal proof of this statement.
- [59] H. M. Fried and D. R. Yennie, “New techniques in the Lamb shift calculation,” Phys. Rev. **112**, 1391 (1958).
- [60] M. I. Eides, H. Grotch, and V. A. Shelyuto, “Theory of light hydrogen-like atoms,” Phys. Rept. **342**, 63 (2001), arXiv:hep-ph/0002158 [hep-ph].
- [61] M. B. Fröb and A. Higuchi, “Mode-sum construction of the two-point functions for the Stueckelberg vector fields in the Poincaré patch of de Sitter space,” J. Math. Phys. **55**, 062301 (2014), arXiv:1305.3421 [gr-qc].
- [62] “NIST Digital Library of Mathematical Functions,” <http://dlmf.nist.gov> (2015).
- [63] S. Deser and R. I. Nepomechie, “Gauge invariance versus masslessness in de Sitter Space,” Annals Phys. **154**, 396 (1984).
- [64] G. W. Gibbons and S. W. Hawking, “Cosmological event horizons, thermodynamics, and particle creation,” Phys. Rev. D **15**, 2738 (1977).
- [65] A. Higuchi and S. S. Kouris, “Large distance behavior of the graviton two point function in de Sitter space-time,” Class. Quant. Grav. **17**, 3077 (2000), arXiv:gr-qc/0004079 [gr-qc].
- [66] B. Allen and T. Jacobson, “Vector two-point functions in maximally symmetric spaces,” Commun. Math. Phys. **103**, 669 (1986).
- [67] S. P. Miao, N. C. Tsamis, and R. P. Woodard, “Transforming to Lorentz gauge on de Sitter,” J. Math. Phys. **50**, 122502 (2009), arXiv:0907.4930 [gr-qc].
- [68] S. Hollands, A. Ishibashi, and D. Marolf, “Comparison between various notions of conserved charges in asymptotically AdS spacetimes,” Class. Quant. Grav. **22**, 2881–2920 (2005), arXiv:hep-th/0503045 [hep-th].
- [69] S. Hollands, A. Ishibashi, and D. Marolf, “Counter-term charges generate bulk symmetries,” Phys. Rev. D **72**, 104025 (2005), arXiv:hep-th/0503105 [hep-th].
- [70] G. Barnich and C. Troessaert, “BMS charge algebra,” JHEP **12**, 105 (2011), arXiv:1106.0213 [hep-th].
- [71] A. Strominger, “On BMS invariance of gravitational scattering,” JHEP **07**, 152 (2014), arXiv:1312.2229 [hep-th].
- [72] T. He, V. Lysov, P. Mitra, and A. Strominger, “BMS supertranslations and Weinberg’s soft graviton theorem,” JHEP **05**, 151 (2015), arXiv:1401.7026 [hep-th].
- [73] J. L. Friedman, “Generic instability of rotating relativistic stars,” Commun. Math. Phys. **62**, 247 (1978).